

A Doubly Corrected Robust Variance Estimator for Linear GMM

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Abstract

We propose a new finite sample corrected variance estimator for the linear generalized method of moments (GMM) including the one-step, two-step, and iterated estimators. Our formula also corrects the over-identification bias in variance estimation on top of the commonly used finite sample correction of Windmeijer (2005), which corrects the bias from estimating the efficient weight matrix, so is *doubly* corrected. An important feature of the proposed double correction is that it automatically provides robustness to misspecification of the moment condition. In contrast, the conventional variance estimator and the Windmeijer correction are inconsistent under misspecification. That is, the double correction formula proposed in this paper provides a convenient way to obtain improved inference under correct specification and robustness against misspecification at the same time.

1 Introduction

The generalized method of moments (GMM) estimators (Hansen, 1982) are widely used in economics. Among the class of GMM estimators, the efficient GMM has the smallest asymptotic variance, which can be obtained via a two-step procedure. However, researchers have found that the standard error of the two-step efficient GMM is often severely downward biased. To solve this problem, Windmeijer (2005) proposed a finite sample bias-corrected standard error formula for the

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two-step linear GMM. Specifically, his formula corrects the bias arising from using the efficient weight matrix being evaluated at an estimate, instead of the true value. The correction formula (the Windmeijer correction, hereafter) has been routinely used in practice.¹

However, the Windmeijer correction does not take into account the *over-identification bias*, which is another important source of bias in the GMM standard error. The over-identification bias arises from the fact that the over-identified sample moment condition is nonzero in general while it converges in probability to zero under correct specification.

We propose a new finite sample correction, which takes into account the over-identification bias for the variance of the linear one-step, two-step, and iterated GMM estimators. For one-step GMM, such as the two-stage least squares (2SLS) estimators, the proposed finite sample correction is new as the Windmeijer correction does not cover the one-step GMM. For two-step and iterated GMM, the proposed correction improves upon the Windmeijer correction by additionally correcting for the over-identification bias. Thus, we *doubly* correct the finite sample bias of the linear GMM variance estimator.

The order of our double correction terms is equal to the order of the sample moment condition. Under correct specification or local misspecification (where the population moment condition is modeled as a drifting sequence within a $n^{-1/2}$ -neighborhood of zero), these terms are $O_p(n^{-1/2})$ so that the double correction becomes a finite-sample correction for the variance. Readers are also referred to a working paper version of this paper (Hwang, Kang, and Lee, 2020), which provides a stochastic expansion of the GMM estimators under local misspecification and shows that the double correction terms estimate the (co)variance of higher-order terms that increase with the over-identification bias.

Under (global) misspecification, however, the double correction terms no longer degenerate to zero asymptotically because the stochastic order of the sample moment condition becomes $O_p(1)$. The conventional variance estimator and the Windmeijer correction omit these $O_p(1)$ terms under misspecification. This implies that the conventional variance estimator and the Windmeijer correction are inconsistent, while our doubly corrected variance estimator is consistent regardless of whether the moment condition model is (locally or globally) misspecified or not.

Since the doubly corrected variance estimators are robust to misspecification, it is not surprising that the formulas coincide with the misspecification-robust variance estimator in Lee (2014) for one-step and two-step GMM and Hansen and Lee (2020) for iterated GMM. Indeed the simulation results reported in those papers show that the misspecification-robust variance estimator often performs better than the conventional sandwich variance estimator under correct specification. This paper provides an answer to this seemingly puzzling result by taking an alternative path to obtain the misspecification-robust variance estimator formula. Our approach provides new insight into the misspecification-robust formula as a finite-sample correction closely related to the well-known Windmeijer (2005) correction. We show that the misspecification-robust variance estimators of Lee (2014) and Hansen and Lee (2020) provide the same order of finite-sample correction with the

¹More than 5,200 citations according to Google Scholar on May 26, 2020.

Windmeijer (2005) correction under correct specification and local misspecification. To the best of our knowledge, this paper is the first to show the equivalence between the finite-sample corrected variance formula and the robust variance formula in misspecified GMM.

From a practical point of view, our paper’s finding implies that accurate inference under correct specification and robust inference under misspecification can be achieved simultaneously, without knowing whether the moment condition is correctly specified. Moreover, practitioners can easily implement bootstrapping the t statistic standardized by the doubly corrected variance estimator and obtain more accurate t -tests and confidence intervals with smaller errors in the size and the coverage. Lee (2014) shows that this bootstrap procedure is robust to misspecification and does not require an ad hoc correction in the bootstrap sample called recentering.

The finite sample correction of the proposed formula and the Windmeijer formula work for linear models. For nonlinear models, the order of the remainder term is the same as the correction terms, so that the corrections do not necessarily provide improvements under correct specification.

Robust inference with possibly misspecified moment condition models has gained considerable attention in the literature. For linear instrumental variable (IV) models, Maasoumi and Phillips (1982) investigate the limiting distribution of inconsistent IV estimators. Guggenberger (2012) studies the behavior of the weak instrument robust tests under local misspecification. Kang (2018) derives higher-order expansions of IV estimators allowing for local violations of the instrument validity condition. Lee (2018) shows that the moment condition is misspecified under treatment effect heterogeneity and proposes a robust variance estimator for 2SLS.

More generally, Hall and Inoue (2003) derive the asymptotic distribution of GMM in misspecified moment condition models. Schennach (2007) proposes an alternative GEL-type estimator robust to global misspecification. Ai and Chen (2007) investigate the asymptotic properties of the sieve minimum distance estimator under misspecified conditional moment restrictions model. Otsu (2011) analyses moderate deviation behaviors of GMM. Kitamura, Otsu, and Evdokimov (2013) propose an estimator that achieves optimal minimax robust properties under local misspecification. Lee (2014, 2016) propose a robust nonparametric bootstrap procedure for GMM and GEL estimators. Hansen and Lee (2020) provide robust inference theory for iterated GMM. Rotemberg (1983) and Andrews (2019) characterize the estimands of the linear GMM under misspecification. Andrews, Gentzkow, and Shapiro (2017) propose to measure the effect of model misspecification on the sensitivity of parameter estimates for the minimum distance estimators. Bonhomme and Weidner (2018) and Armstrong and Kolesár (2019) consider minimax and GMM inference under possible misspecification, respectively.

Finite sample properties of GMM estimators, including the iterated and the continuously updating (CU) GMM, are investigated by Hansen, Heaton, and Yaron (1996). Bond and Windmeijer (2005) provide simulation evidence on the finite sample performance of the asymptotic and bootstrap tests based on GMM estimators. Hwang (2020) develops fixed-cluster asymptotics and finite-sample corrected variance formula for cross-sectionally dependent data. Hwang and Sun (2018) employ fixed-smoothing asymptotics to provide a more accurate comparison between the one-step

and two-step GMM procedures for time-series observations.

Our doubly corrected robust variance estimators are generally different from the many instruments and many weak instruments robust variance estimators for IV, GMM, and GEL, e.g., Bekker (1994), Han and Phillips (2006), Newey and Windmeijer (2009), and Evdokimov and Kolesár (2018). Since our double correction formula does not use the many (weak) instruments asymptotics, it is not robust under such sequences.

The remainder of the paper is organized as follows. Section 2 reviews the Windmeijer correction. Section 3 proposes the doubly corrected variance estimator. Section 4 shows that the doubly corrected variance estimator is misspecification-robust. Section 5 discusses the iterated GMM and the CU GMM estimators. Section 6 derives the double correction formula for cross-sectional IV and panel data with the difference GMM. Section 7 provides extensive simulation results comparing the double correction and other variance estimators. Finally, Section 8 concludes. All the proofs are collected in the Appendix.

2 Finite Sample Correction of Windmeijer (2005)

Suppose that we observe a sequence of i.i.d. random vectors $X_i \in \mathbb{R}^{d_x}$ for $i = 1, \dots, n$. Let $g(X_i, \theta)$ be a $q \times 1$ moment function where θ is a $k \times 1$ parameter vector. We assume $q > k$ so that the model is over-identified and $g(X_i, \theta)$ is linear in parameter. When the model is just-identified ($q = k$) the correction terms are zeros and the analysis becomes trivial. The moment condition model is correctly specified if

$$E[g(X_i, \theta_0)] = 0 \tag{1}$$

for the true value θ_0 . Assume $E[\|g(X_i, \theta_0)\|^2] < \infty$ so that

$$\frac{1}{n} \sum_{i=1}^n g(X_i, \theta_0) \equiv g_n(\theta_0) = O_p(n^{-1/2}).$$

Thus, the sample moment condition converges in probability to zero at the rate of $n^{-1/2}$ under correct specification (1). This will be used in determining the order of higher-order terms in Sections 2 and 3.

The one-step GMM estimator is defined as

$$\hat{\theta}_1 = \arg \min_{\theta \in \Theta} g_n(\theta)' W_n^{-1} g_n(\theta),$$

where W_n is a $q \times q$ positive definite weight matrix which takes the form of $n^{-1} \sum_{i=1}^n W(X_i)$ and $W(X_i)$ does not depend on any unknown parameter. Common choices of $W(X_i)$ are the identity matrix and $Z_i Z_i'$, where Z_i is the instrument vector in IV regressions. Let $W = E W_n$, a positive definite matrix of constants.

The two-step efficient GMM estimator using $\hat{\theta}_1$ as a preliminary (initial) estimator is defined as

$$\hat{\theta}_2 = \arg \min_{\theta \in \Theta} g_n(\theta)' [\Omega_n(\hat{\theta}_1)]^{-1} g_n(\theta),$$

where

$$\Omega_n(\theta) = \frac{1}{n} \sum_{i=1}^n g(X_i, \theta) g(X_i, \theta)'$$

Define $\Omega = \Omega(\theta_0)$ where $\Omega(\theta) = E\Omega_n(\theta)$. Since $\Omega_n(\hat{\theta}_1)$ is consistent for the asymptotic variance of the moment function, the two-step GMM estimator is efficient.

We also define an infeasible two-step GMM estimator $\tilde{\theta}_2$ using $[\Omega_n(\theta_0)]^{-1}$ as the weight matrix:

$$\tilde{\theta}_2 = \arg \min_{\theta \in \Theta} g_n(\theta)' [\Omega_n(\theta_0)]^{-1} g_n(\theta).$$

Investigating the limiting behavior of $\sqrt{n}(\tilde{\theta}_2 - \theta_0)$ will help us understand the higher-order behavior of the feasible two-step estimator $\sqrt{n}(\hat{\theta}_2 - \theta_0)$.

Let $G(X_i) = \partial g(X_i, \theta) / \partial \theta'$. Note that it does not depend on θ due to the linearity. Define $G_n = n^{-1} \sum_{i=1}^n G(X_i)$ and $G = EG_n$, which is assumed full column-rank. By the first-order Taylor expansion, the first-order condition (FOC) of the (feasible) two-step GMM estimator can be written as

$$0 = G_n' [\Omega_n(\hat{\theta}_1)]^{-1} g_n(\hat{\theta}_2) = G_n' [\Omega_n(\hat{\theta}_1)]^{-1} [g_n(\theta_0) + G_n(\hat{\theta}_2 - \theta_0)], \quad (2)$$

and so we have

$$\sqrt{n}(\hat{\theta}_2 - \theta_0) = - \left\{ G_n' [\Omega_n(\hat{\theta}_1)]^{-1} G_n \right\}^{-1} G_n' [\Omega_n(\hat{\theta}_1)]^{-1} \sqrt{n} g_n(\theta_0). \quad (3)$$

Using a similar expansion, we can get

$$\sqrt{n}(\tilde{\theta}_2 - \theta_0) = - \left\{ G_n' [\Omega_n(\theta_0)]^{-1} G_n \right\}^{-1} G_n' [\Omega_n(\theta_0)]^{-1} \sqrt{n} g_n(\theta_0) \quad (4)$$

for the infeasible two-step GMM estimator, and

$$\sqrt{n}(\hat{\theta}_1 - \theta_0) = -(G_n' W_n^{-1} G_n)^{-1} G_n' W_n^{-1} \sqrt{n} g_n(\theta_0) \quad (5)$$

for the one-step GMM estimator.

Asymptotically (3) and (4) have the same limiting distribution so that using $\Omega_n(\hat{\theta}_1)$ instead of $\Omega_n(\theta_0)$ does not affect the first-order asymptotic analysis. However, by expanding $\Omega_n(\hat{\theta}_1)$ around θ_0 and using (5), Windmeijer (2005) shows that the extra finite sample variation caused by higher-order terms can be estimated and one can improve the accuracy of the variance estimator for linear moment conditions.

To see this, we use the first-order Taylor expansion of $\Omega_n(\hat{\theta}_1)$ on the right-hand side (RHS) of

(3) around θ_0 :

$$\begin{aligned}\sqrt{n}(\hat{\theta}_2 - \theta_0) &= -\{G'_n[\Omega_n(\theta_0)]^{-1}G_n\}^{-1}G'_n[\Omega_n(\theta_0)]^{-1}\sqrt{n}g_n(\theta_0) + D_n\sqrt{n}(\hat{\theta}_1 - \theta_0) + R_n \\ &= \sqrt{n}(\tilde{\theta}_2 - \theta_0) + \underbrace{D_n\sqrt{n}(\hat{\theta}_1 - \theta_0)}_{=O_p(n^{-1/2})} + R_n,\end{aligned}\tag{6}$$

where

$$\begin{aligned}D_n &= F_{1n} + F_{2n}, \\ F_{1n} &= -\left.\frac{\partial \{G'_n[\Omega_n(\theta)]^{-1}G_n\}^{-1}}{\partial \theta'}\right|_{\theta=\theta_0} G'_n[\Omega_n(\theta_0)]^{-1}g_n(\theta_0), \\ F_{2n} &= -\{G'_n[\Omega_n(\theta_0)]^{-1}G_n\}^{-1}\left.\frac{\partial \{G'_n[\Omega_n(\theta)]^{-1}g_n(\theta)\}}{\partial \theta'}\right|_{\theta=\theta_0}\end{aligned}$$

are $k \times k$ matrices and R_n is the remainder term. Since $g_n(\theta_0) = O_p(n^{-1/2})$ both F_{1n} and F_{2n} are $O_p(n^{-1/2})$. Thus, the second term in the RHS of (6) is of order $O_p(n^{-1/2})$ assuming that $\sqrt{n}(\hat{\theta}_1 - \theta_0) = O_p(1)$. The remainder term R_n is of order $O_p(n^{-1})$ because of the linearity of the moment function, provided that the higher moments of $g(X_i, \theta_0)$ and $G(X_i)$ exist (the formal justification is given in the proof of Theorem 1). Thus, by taking into account for the variation caused by the $O_p(n^{-1/2})$ term, the finite sample variance of $\sqrt{n}(\hat{\theta}_2 - \theta_0)$ can be more accurately approximated. Note that the expansion (6) only holds for linear moment condition models.

The Windmeijer correction of the variance of $\sqrt{n}(\hat{\theta}_2 - \theta_0)$ is obtained by

$$\widehat{V}_w(\hat{\theta}_2) = \widetilde{V}(\hat{\theta}_2) + \widehat{D}_n\widetilde{V}(\hat{\theta}_2) + \widetilde{V}(\hat{\theta}_2)\widehat{D}'_n + \widehat{D}_n\widetilde{V}(\hat{\theta}_1)\widehat{D}'_n,\tag{7}$$

where $D[\cdot, j]$ denotes the j th column of D , $\theta_{[j]}$ denotes the j th element of θ , and

$$\begin{aligned}\widetilde{V}(\hat{\theta}_1) &= (G'_n W_n^{-1} G_n)^{-1} \left(G'_n W_n^{-1} \Omega_n(\hat{\theta}_1) W_n^{-1} G_n \right) (G'_n W_n^{-1} G_n)^{-1}, \\ \widetilde{V}(\hat{\theta}_2) &= \left\{ G'_n [\Omega_n(\hat{\theta}_1)]^{-1} G_n \right\}^{-1}, \\ \widehat{D}_n[\cdot, j] &= \left\{ G'_n [\Omega_n(\hat{\theta}_1)]^{-1} G_n \right\}^{-1} G'_n \left\{ [\Omega_n(\hat{\theta}_1)]^{-1} \frac{\partial \Omega_n(\theta)}{\partial \theta_{[j]}} \Big|_{\theta=\hat{\theta}_1} [\Omega_n(\hat{\theta}_1)]^{-1} \right\} g_n(\hat{\theta}_2), \\ \frac{\partial \Omega_n(\theta)}{\partial \theta_{[j]}} &= \Upsilon_j(\theta) + \Upsilon'_j(\theta), \\ \Upsilon_j(\theta) &= \frac{1}{n} \sum_{i=1}^n g(X_i, \theta) \frac{\partial g(X_i, \theta)'}{\partial \theta_{[j]}}.\end{aligned}$$

Since the estimate of F_{1n} equals to zero because of the FOC, $0 = G'_n[\Omega_n(\hat{\theta}_1)]^{-1}g_n(\hat{\theta}_2)$, it does not appear in the variance estimator formula. The standard error is obtained by taking the diagonal elements of $\sqrt{\widehat{V}_w(\hat{\theta}_2)/n}$.

3 Double Correction

The Windmeijer correction accounts for the extra variability due to using an estimated parameter in the weight matrix. This correction is effective because $\widehat{D}_n \neq 0$, which is due to $g_n(\hat{\theta}_2) \neq 0$ in finite sample. In fact, $g_n(\theta) \neq 0$ for all θ almost surely if at least one of the moments is continuously distributed, which trivially implies $g_n(\theta_0) \neq 0$. We call this the *over-identification bias*, which is non-zero for any n in general for over-identified models.

We show that the over-identification bias causes additional finite sample variability in (6). These additional terms are not considered in the Windmeijer correction (7). We propose alternative variance estimators that fully incorporate the additional variation induced by the over-identification bias. These variance estimators will replace $\tilde{V}(\hat{\theta}_2)$ and $\tilde{V}(\hat{\theta}_1)$ in (7) without affecting the order of finite sample corrections, leading to our doubly corrected variance estimator.

Define $g = g(\theta_0)$ where $g(\theta) = E[g(X_i, \theta)]$. Assume that

$$G_n - G = O_p(n^{-1/2}), \quad (8)$$

$$\text{vec}(W_n - W) = O_p(n^{-1/2}), \quad (9)$$

$$\text{vec}(\Omega_n(\theta_0) - \Omega) = O_p(n^{-1/2}), \quad (10)$$

which hold under appropriate regularity conditions. Since $G'\Omega^{-1}g = 0$ by the population FOC and

$$[\Omega_n(\theta_0)]^{-1} - \Omega^{-1} = -\Omega^{-1}(\Omega_n(\theta_0) - \Omega)[\Omega_n(\theta_0)]^{-1},$$

we can write

$$\begin{aligned} & G'_n[\Omega_n(\theta_0)]^{-1}g_n(\theta_0) \\ &= \underbrace{G'\Omega^{-1}g_n(\theta_0)}_{=O_p(n^{-1/2})} + \underbrace{(G_n - G')'\Omega^{-1}g_n(\theta_0) - G'\Omega^{-1}(\Omega_n(\theta_0) - \Omega)\Omega^{-1}g_n(\theta_0)}_{=O_p(n^{-1})} + O_p(n^{-3/2}). \end{aligned} \quad (11)$$

Using (11), the expansion of the infeasible two-step GMM estimator (4) can be written as

$$\sqrt{n}(\tilde{\theta}_2 - \theta_0) = - \{G'_n[\Omega_n(\theta_0)]^{-1}G_n\}^{-1} [G'\Omega^{-1}\sqrt{n}g_n(\theta_0)] \quad (12)$$

$$+ \sqrt{n}(G_n - G')'\Omega^{-1}g_n(\theta_0) - G'\Omega^{-1}\sqrt{n}(\Omega_n(\theta_0) - \Omega)\Omega^{-1}g_n(\theta_0)] \quad (13)$$

$$+ O_p(n^{-1}). \quad (14)$$

Similarly,

$$\sqrt{n}(\hat{\theta}_1 - \theta_0) = - \{G'_n W_n^{-1} G_n\}^{-1} [G' W^{-1} \sqrt{n} g_n(\theta_0)] \quad (15)$$

$$+ \sqrt{n}(G_n - G')' W^{-1} g_n(\theta_0) - G' W^{-1} \sqrt{n} (W_n - W) W^{-1} g_n(\theta_0)] \quad (16)$$

$$+ O_p(n^{-1}), \quad (17)$$

which simplifies to

$$\sqrt{n}(\hat{\theta}_1 - \theta_0) = -\{G'_n G_n\}^{-1} [G' \sqrt{n} g_n(\theta_0) + \sqrt{n}(G_n - G)' g_n(\theta_0)]$$

when $W_n = I$. From the above expansions, we learn the followings. First, we need to consider the extra variation from $\sqrt{n}(G_n - G)$ and $\sqrt{n}(\Omega_n(\theta) - \Omega)$ (or $\sqrt{n}(W_n - W)$ for the one-step GMM) to account for the over-identification bias. Second, the order of the remainder term of the original expansion (6) is not changed.

Using the expansions (12)-(14) and (15)-(17), the expansion of the two-step GMM can be written as

$$\sqrt{n}(\hat{\theta}_2 - \theta_0) = -\{G'_n[\Omega_n(\theta_0)]^{-1} G_n\}^{-1} [G' \Omega^{-1} \sqrt{n} g_n(\theta_0)] \quad (18)$$

$$+ \sqrt{n}(G_n - G)' \Omega^{-1} g_n(\theta_0) - G' \Omega^{-1} \sqrt{n} (\Omega_n(\theta_0) - \Omega) \Omega^{-1} g_n(\theta_0)] \quad (19)$$

$$- D_n \{G'_n W_n^{-1} G_n\}^{-1} [G' W^{-1} \sqrt{n} g_n(\theta_0)] \quad (20)$$

$$+ \sqrt{n}(G_n - G)' W^{-1} g_n(\theta_0) - G' W^{-1} \sqrt{n} (W_n - W) W^{-1} g_n(\theta_0)] \quad (21)$$

$$+ O_p(n^{-1}). \quad (22)$$

Note that (22) is the sum of (14), (17), and R_n in (6). The first two terms are $O_p(n^{-1})$ under (8)-(10).

In finite sample, $g_n(\theta_0) \neq 0$ because $g_n(\theta) \neq 0$ for all θ almost surely, and this causes extra variation through the terms in (19) and (21). Similar to the Windmeijer correction, by taking into account for these (asymptotically negligible) terms in estimating the variance we can make more accurate inference.

Since $D_n = O_p(n^{-1/2})$, the terms in (21) multiplied by D_n are $O_p(n^{-1})$, which is the same order as the remainder term. Thus, considering those terms in (21) does not necessarily provide finite sample corrections. However, including these terms are critical to getting robustness to misspecification, which is shown in Section 4.

The expansion for the one-step GMM is (15)-(17), and those terms in (16) are $O_p(n^{-1/2})$. Thus, considering the finite sample variation caused by these terms provides a more accurate variance estimator formula. This correction for the one-step GMM is not considered in Windmeijer (2005) and is new.

The doubly corrected variance estimator of $\sqrt{n}(\hat{\theta}_2 - \theta_0)$ is

$$\hat{V}_{dc}(\hat{\theta}_2) = \hat{V}(\hat{\theta}_2) + \hat{D}_n \hat{C}(\hat{\theta}_1, \hat{\theta}_2) + \hat{C}(\hat{\theta}_1, \hat{\theta}_2)' \hat{D}'_n + \hat{D}_n \hat{V}_{dc}(\hat{\theta}_1) \hat{D}'_n,$$

where

$$\begin{aligned}\widehat{V}(\hat{\theta}_2) &= \left(G'_n[\Omega_n(\hat{\theta}_1)]^{-1}G_n\right)^{-1} \Sigma_n(\hat{\theta}_2, \Omega_n(\hat{\theta}_1)) \left(G'_n[\Omega_n(\hat{\theta}_1)]^{-1}G_n\right)^{-1}, \\ \widehat{V}_{dc}(\hat{\theta}_1) &= (G'_n W_n^{-1} G_n)^{-1} \Sigma_n(\hat{\theta}_1, W_n) (G'_n W_n^{-1} G_n)^{-1}, \\ \widehat{C}(\hat{\theta}_1, \hat{\theta}_2) &= (G'_n W_n^{-1} G_n)^{-1} \frac{1}{n} \sum_{i=1}^n m_i(\hat{\theta}_1, W_n) m_i(\hat{\theta}_2, \Omega_n(\hat{\theta}_1))' \left(G'_n[\Omega_n(\hat{\theta}_1)]^{-1}G_n\right)^{-1},\end{aligned}$$

and

$$\begin{aligned}\Sigma_n(\theta, \Xi_n(\phi)) &= \frac{1}{n} \sum_{i=1}^n m_i(\theta, \Xi_n(\phi)) m_i(\theta, \Xi_n(\phi))', \\ m_i(\theta, \Xi_n(\phi)) &= G'_n[\Xi_n(\phi)]^{-1} g(X_i, \theta) + G(X_i)'[\Xi_n(\phi)]^{-1} g_n(\theta) \\ &\quad - G'_n[\Xi_n(\phi)]^{-1} \Xi(X_i, \phi) [\Xi_n(\phi)]^{-1} g_n(\theta), \\ \Xi_n(\phi) &= \frac{1}{n} \sum_{i=1}^n \Xi(X_i, \phi).\end{aligned}\tag{23}$$

When $\Xi_n(\phi) = \Xi(X_i, \phi) = I$, the last term of $m(\theta, \Xi_n(\phi))$ drops. Note that $G(X_i)$ and $\Xi(X_i, \phi)$ in $m_i(\theta, \Xi_n(\phi))$ are not centered because the FOCs hold evaluated at $(\hat{\theta}_2, \Omega_n(\hat{\theta}_1))$ and $(\hat{\theta}_1, W_n)$, respectively.

The doubly corrected variance estimator for the two-step GMM, $\widehat{V}_{dc}(\hat{\theta}_2)$, provides the same order of finite sample correction as the Windmeijer correction, $\widehat{V}_w(\hat{\theta}_2)$. The standard error is obtained by taking the diagonal elements of $\sqrt{\widehat{V}_{dc}(\hat{\theta}_2)/n}$.

The doubly corrected variance estimator for the one-step GMM, $\widehat{V}_{dc}(\hat{\theta}_1)$, accounts for the variations up to the order of $O_p(n^{-1/2})$ in the expansion (15)-(17). This correction is not considered in Windmeijer (2005). The standard error is obtained by taking the diagonal elements of $\sqrt{\widehat{V}_{dc}(\hat{\theta}_1)/n}$.

4 Robustness to Misspecification

The variance estimators considered so far, the doubly corrected, the Windmeijer corrected, and the conventional, are consistent for the asymptotic variance of $\sqrt{n}(\hat{\theta}_2 - \theta_0)$ under correct specification, $E[g(X_i, \theta_0)] = 0$. In words, the correct specification means that an over-identified model exactly holds at the true value θ_0 , but this may be too restrictive in reality. Indeed, the sample moment condition does not hold for any finite sample size n almost surely if the model is over-identified, i.e., $g_n(\hat{\theta}) \neq 0$, provided that at least one of the moments is continuously distributed. Thus, it is reasonable to view the assumed moment condition model as the best-approximating model and to allow for possible misspecification.

Under (global) misspecification, which is defined as

$$E[g(X_i, \theta)] = \delta(\theta) \neq 0, \quad \forall \theta \in \Theta,$$

where $\delta(\theta)$ is a vector of constants and Θ is the parameter space, the GMM estimator is consistent for the pseudo-true value which is defined as the unique minimizer of the population GMM criterion given the weight matrix (Hall and Inoue, 2003). In addition, the asymptotic variance has more terms that are assumed away under correct specification. Thus, the conventional variance estimators are no longer consistent under misspecification. Lee (2014) proposes variance estimators for one-step and two-step GMM under misspecification. Hansen and Lee (2019) propose a similar robust variance estimator for iterated GMM. These variance estimators are shown to be consistent regardless of misspecification and they are referred to as the misspecification-robust variance estimator, hereafter.

Since the misspecification-robust variance estimators contain additional terms that are not present in the conventional variance estimator, one may conjecture that it is less accurate than the conventional formula under correct specification. We show that this conjecture is not true by proving that the doubly corrected variance estimator $\widehat{V}_{dc}(\hat{\theta}_2)$ is the misspecification-robust variance estimator.

To formally show that $\widehat{V}_{dc}(\hat{\theta}_2)$ is consistent for the asymptotic variance under misspecification, we introduce additional definitions. Define the one-step and two-step GMM (pseudo-) true values as

$$\theta_1 = \arg \min_{\theta \in \Theta} E[g(X_i, \theta)]' W^{-1} E[g(X_i, \theta)], \quad (24)$$

$$\theta_2 = \arg \min_{\theta \in \Theta} E[g(X_i, \theta)]' [\Omega(\theta_1)]^{-1} E[g(X_i, \theta)]. \quad (25)$$

In general $\theta_1 \neq \theta_2$, but $\theta_1 = \theta_2 = \theta_0$ under correct specification. Write $g_j = E[g(X_i, \theta_j)]$ and $\Omega_j = \Omega(\theta_j)$ for $j = 1, 2$. In the presence of global misspecification, $g_n(\theta_j) = O_p(1)$ for $j = 1, 2$.

The robustness of $\widehat{V}_{dc}(\hat{\theta}_2)$ holds for the following reasons. Recall that the formulas for $\widehat{V}_{dc}(\hat{\theta}_2)$ and $\widehat{V}_w(\hat{\theta}_2)$ are given by

$$\begin{aligned} \widehat{V}_{dc}(\hat{\theta}_2) &= \widehat{V}(\hat{\theta}_2) + \widehat{D}_n \widehat{C}(\hat{\theta}_1, \hat{\theta}_2) + \widehat{C}(\hat{\theta}_1, \hat{\theta}_2)' \widehat{D}_n' + \widehat{D}_n \widehat{V}_{dc}(\hat{\theta}_1) \widehat{D}_n', \\ \widehat{V}_w(\hat{\theta}_2) &= \widetilde{V}(\hat{\theta}_2) + \widehat{D}_n \widetilde{V}(\hat{\theta}_2) + \widetilde{V}(\hat{\theta}_2) \widehat{D}_n' + \widehat{D}_n \widetilde{V}(\hat{\theta}_1) \widehat{D}_n'. \end{aligned}$$

The correction term $\widehat{D}_n$ corrects the bias in the variance due to using the weight matrix $[\Omega_n(\hat{\theta}_1)]^{-1}$ instead of $[\Omega_n(\theta_1)]^{-1}$. Since both $\widehat{V}_{dc}(\hat{\theta}_2)$ and $\widehat{V}_w(\hat{\theta}_2)$ have $\widehat{D}_n$, this bias is corrected in both variance estimators. What is not accounted for in the Windmeijer corrected variance estimator is the additional variation in the sample Jacobian G_n , and the sample weight matrices W_n and $\Omega_n(\theta_1)$. These variations are asymptotically negligible under correct specification but become the first-order under misspecification. Our doubly corrected variance estimator $\widehat{V}_{dc}(\hat{\theta}_2)$ accounts for these variations.

The expansion of the GMM estimators in misspecified models is quite similar to those in correctly specified models, except that we need to allow different pseudo-true values for one-step and two-step GMM, and the moment condition evaluated at the pseudo-true value is not equal to zero.

Consider the FOC of the two-step GMM estimator (2). By expanding $g_n(\hat{\theta}_2)$ around θ_2 , and $\Omega_n(\hat{\theta}_1)$ around θ_1 , we can write

$$\begin{aligned}\sqrt{n}(\hat{\theta}_2 - \theta_2) &= - \{G'_n[\Omega_n(\theta_1)]^{-1}G_n\}^{-1} G'_n[\Omega_n(\theta_1)]^{-1}\sqrt{n}g_n(\theta_2) + D_n^*\sqrt{n}(\hat{\theta}_1 - \theta_1) + R_n^* \\ &= \sqrt{n}(\tilde{\theta}_2^* - \theta_2) + D_n^*\sqrt{n}(\hat{\theta}_1 - \theta_1) + R_n^*,\end{aligned}\quad (26)$$

where $\tilde{\theta}_2^*$ is defined as

$$\tilde{\theta}_2^* = \arg \min_{\theta \in \Theta} g_n(\theta)'[\Omega_n(\theta_1)]^{-1}g_n(\theta), \quad (27)$$

and

$$\begin{aligned}D_n^* &= F_{1n}^* + F_{2n}^*, \\ F_{1n}^* &= - \left. \frac{\partial \{G'_n[\Omega_n(\theta)]^{-1}G_n\}}{\partial \theta'} \right|_{\theta=\theta_1} G'_n[\Omega_n(\theta_1)]^{-1}g_n(\theta_2), \\ F_{2n}^* &= - \{G'_n[\Omega_n(\theta_1)]^{-1}G_n\}^{-1} \left. \frac{\partial G'_n[\Omega_n(\theta)]^{-1}g_n(\theta_2)}{\partial \theta'} \right|_{\theta=\theta_1},\end{aligned}$$

and R_n^* is the remainder term of order $O_p(n^{-1/2}\|g_n(\theta_2)\|)$ (which is formally justified with the order of other terms in the proof of Theorem 1). Since $D_n^* = O_p(\|g_n(\theta_2)\|)$, the order of finite sample correction depends on the degree of misspecification, from being $O_p(n^{-1/2})$ under correct specification to $O_p(1)$ under misspecification. Note that both $\sqrt{n}(\tilde{\theta}_2^* - \theta_2)$ and $D_n^*\sqrt{n}(\hat{\theta}_1 - \theta_1)$ in (26) are $O_p(1)$ under misspecification and this will alter the first-order asymptotic variance.

Using the population FOC $G'\Omega_j^{-1}g_j = 0$ for $j = 1, 2$, the FOC of the infeasible two-step GMM estimator (27) can be expanded as

$$\begin{aligned}\sqrt{n}(\tilde{\theta}_2^* - \theta_2) &= - \{G'_n[\Omega_n(\theta_1)]^{-1}G_n\}^{-1} \{G'_n[\Omega_n(\theta_1)]^{-1}\sqrt{n}(g_n(\theta_2) - g_2) \\ &\quad + \sqrt{n}(G_n - G)'[\Omega_n(\theta_1)]^{-1}g_2 - G'\Omega_1^{-1}\sqrt{n}(\Omega_n(\theta_1) - \Omega_1)[\Omega_n(\theta_1)]^{-1}g_2\}.\end{aligned}\quad (28)$$

The FOC of the one-step GMM can be expanded similarly:

$$\begin{aligned}\sqrt{n}(\hat{\theta}_1 - \theta_1) &= - \{G'_n W_n^{-1}G_n\}^{-1} [G'_n W_n^{-1}\sqrt{n}(g_n(\theta_1) - g_1) \\ &\quad + \sqrt{n}(G_n - G)'W_n^{-1}g_1 - G'W^{-1}\sqrt{n}(W_n - W)W_n^{-1}g_1].\end{aligned}\quad (29)$$

The expansions (28) and (29) are misspecification-robust versions of (4) and (5), allowing for different probability limits of the one-step and the two-step GMM estimators, and taking into account for the misspecification (over-identification) bias. Under correct specification, $g_1 = g_2 = 0$ and (28) and (29) coincide with (4) and (5).

Now we list assumptions for the main result.

Assumption 1.

- (i) θ_j is unique and is in the interior of the parameter space Θ for $j = 1, 2$

(ii) $X_1, \dots, X_n$ are i.i.d.

(iii) W and $\Omega(\theta_1)$ are nonsingular

(iv) G is full column rank

(v) $E[\|g(X_i, \theta_1)\|^4] < \infty$, $E[\|g(X_i, \theta_2)\|^2] < \infty$, $E[\|G(X_i)\|^4] < \infty$, $E[\|W(X_i)\|^2] < \infty$

Assumption 1 is mild regularity conditions for consistency and asymptotic normality of the one-step and the two-step linear GMM estimators allowing for global misspecification. For misspecified models, Hall and Inoue (2003) provide a list of conditions for one-step and two-step GMM in the time series context. Hansen and Lee (2020) provide a list of conditions for one-step and iterated GMM under the independent but not necessarily identically distributed (i.n.i.d.) sampling.

The following theorem shows that the doubly corrected variance estimators of one-step and two-step linear GMM are consistent for the asymptotic variance matrices under misspecification. The proof is given in the Appendix A.

Theorem 1. *Suppose that Assumption 1 holds. As $n \rightarrow \infty$, for $j = 1, 2$,*

$$\sqrt{n}(\hat{\theta}_j - \theta_j) \xrightarrow{d} N(0, V_j)$$

and

$$\widehat{V}_{dc}(\hat{\theta}_j) \xrightarrow{p} V_j.$$

Theorem 1 holds regardless of whether the model is correctly specified or not. Thus, $\widehat{V}_{dc}(\hat{\theta}_j)$ for $j = 1, 2$, provides a finite sample correction under correct specification and it remains consistent under misspecification. In contrast, the Windmeijer corrected variance estimator, $\widehat{V}_w(\hat{\theta}_2)$, is not first-order consistent under misspecification. Assuming the linearity of moment condition, the proofs of Lemma 2 and Theorem 1 in Appendix A can be used to rigorously justify the stochastic orders of the higher-order terms mentioned in the previous sections.

For nonlinear models, $\widehat{V}_{dc}(\hat{\theta}_j)$ formula can be adjusted by considering the second derivative of the moment function, which coincides with the misspecification-robust formula of Lee (2014) and Hansen and Lee (2020). The same consistency result with Theorem 1 is shown in those papers, but stronger assumptions on the moment/Jacobian processes and the compact parameter space are required to use the uniform law of large numbers.

Theorem 1 also implies that the nonparametric i.i.d. bootstrap t test and confidence intervals (CIs) based on the GMM t statistic studentized with the doubly corrected standard error automatically achieve higher-order refinements over the asymptotic t test and CIs regardless of misspecification (Lee, 2014). In contrast, those bootstrap t test and CIs based on the GMM t statistic studentized with the conventional or the Windmeijer standard error require an additional recentering procedure in resampling to correct for the over-identification bias to achieve higher-order

refinements, see Hall and Horowitz (1996) and Andrews (2002). Furthermore, the conventional nonparametric i.i.d. bootstrap procedure for GMM is not valid under misspecification. Thus, our doubly corrected variance estimator formula provides a very convenient way to get more accurate but also robust bootstrap tests and CIs.

Remark 1 (Stochastic expansion under local misspecification) From the way it is constructed, we can directly see that the doubly corrected variance formulas provide a finite-sample variance correction with the same argument as Windmeijer (2005). It is expected to work the best when the sample moment condition is large. Since a nonzero sample moment condition can also be due to a locally misspecified moment condition, one can seek an additional justification of the double correction by deriving the stochastic expansions of the GMM estimators under such a sequence. In a working paper version of this paper (Hwang, Kang, and Lee, 2020), we provide formal stochastic expansions of the one-step and the two-step GMM estimators by allowing the population moment condition evaluated at the true value is $E[g(X_{in}, \theta_0)] = \delta/\sqrt{n}$ for some $\delta \neq 0$. We also discuss how the first-order terms D_n^* , (28), and (29) in the double correction are related to the higher-order terms under local misspecification. Interestingly, the analysis reveals that the double correction effectively estimates the (co)variances of higher-order terms that depend on δ and thus is fully robust to additional variations due to local misspecification. In contrast, the Windmeijer corrected variance estimator only partially considers the higher-order terms that depend on δ , making it only partly robust to local misspecification. Finally, the stochastic expansions show that both the doubly corrected and the Windmeijer corrected variance estimators are not higher-order variance estimators which would estimate additional terms up to $O(n^{-1})$. For the general treatment of the stochastic expansion, see Rothenberg (1984). Newey and Smith (2004) derive the stochastic expansion of GMM and the generalized empirical likelihood (GEL) estimators under correct specification.

Remark 2 (Weight matrix) The main results hold if we replace $\Omega_n(\theta)$ with the centered weight matrix

$$\Omega_n^c(\theta) = \frac{1}{n} \sum_{i=1}^n (g(X_i, \theta) - g_n(\theta)) (g(X_i, \theta) - g_n(\theta))', \quad (30)$$

with some specifics need to be modified accordingly. Specifically, the two-step GMM pseudo-true value θ_2 defined in (25) is now defined with $\Omega_n^c(\theta_1)$. In addition, the derivative of the centered weight matrix is

$$\Upsilon_j^c(\theta) = \frac{1}{n} \sum_{i=1}^n (g(X_i, \theta) - g_n(\theta)) \left(\frac{\partial g(X_i, \theta)}{\partial \theta_{[j]}} - \frac{\partial g_n(\theta)}{\partial \theta_{[j]}} \right)'$$

so that

$$\frac{\partial \Omega_n^c(\theta)}{\partial \theta_{[j]}} = \Upsilon_j^c(\theta) + \Upsilon_j^c(\theta).$$

The centered weight matrix is consistent for the asymptotic variance matrix of the moment equation under misspecification. Hansen (2020) recommends using the centered weight matrix for this reason.

Hall (2000) shows that the GMM over-identification test statistic with a centered heteroskedasticity-and-autocorrelation-consistent (HAC) weight matrix leads to more powerful tests in the time series setting.

5 Iterated GMM and Continuously Updating GMM

Both the Windmeijer and our double correction correct the extra variation due to the weight matrix being evaluated at an estimate rather than the true value. A natural question is whether similar finite sample corrections can be obtained for other GMM estimators, namely the iterated GMM estimator and the continuously-updating (CU) GMM estimator of L. Hansen, Heaton, and Yaron (1996). We show that the answer is yes for the iterated GMM estimator, and the double correction formula is the same as the misspecification-robust formula. For the CU GMM estimator, the answer is negative.

Assume the correct specification. The iterated GMM estimator is obtained by iterating the two-step efficient GMM estimator until convergence. By iteration the dependence of the final estimator on the previous step estimators disappears. The FOC is given by

$$0 = G'_n[\Omega_n(\hat{\theta})]^{-1} g_n(\hat{\theta}),$$

where $\hat{\theta}$ is the iterated GMM estimator. Assume that $g_n(\theta_0) = O_p(n^{-1/2})$ and $\hat{\theta} - \theta_0 = O_p(n^{-1/2})$ whose sufficient conditions are provided in Hansen and Lee (2020). By applying the first-order Taylor expansion around θ_0 to $g_n(\hat{\theta})$ and $\Omega_n(\hat{\theta})$ sequentially

$$\sqrt{n}(\hat{\theta} - \theta_0) = - \{G'_n[\Omega_n(\theta_0)]^{-1} G_n\}^{-1} G'_n[\Omega_n(\theta_0)]^{-1} \sqrt{n} g_n(\theta_0) + D_n \sqrt{n}(\hat{\theta} - \theta_0) + O_p(n^{-1}),$$

and thus

$$\sqrt{n}(\hat{\theta} - \theta_0) = - \{G'_n[\Omega_n(\theta_0)]^{-1} G_n(I_k - D_n)\}^{-1} G'_n[\Omega_n(\theta_0)]^{-1} \sqrt{n} g_n(\theta_0) + O_p(n^{-1}). \quad (31)$$

Windmeijer (2000) proposes a finite sample corrected variance estimator based on the expansion (31). We proceed one additional step. By further expanding to take into account for the over-identification bias, we have

$$\sqrt{n}(\hat{\theta} - \theta_0) = - \{G'_n[\Omega_n(\theta_0)]^{-1} G_n(I_k - D_n)\}^{-1} [G' \Omega^{-1} \sqrt{n} g_n(\theta_0) \quad (32)$$

$$+ \sqrt{n}(G_n - G)' \Omega^{-1} g_n(\theta_0) - G' \Omega^{-1} \sqrt{n} (\Omega_n(\theta_0) - \Omega) \Omega^{-1} g_n(\theta_0)] + O_p(n^{-1}). \quad (33)$$

Since the remainder term is $O_p(n^{-1})$, by estimating the variance of the terms in (32)-(33) up to $O_p(n^{-1/2})$ we can get the same order of finite sample correction with the doubly corrected two-step GMM variance estimator.

The doubly corrected variance estimator for iterated GMM is

$$\widehat{V}_{dc}(\hat{\theta}) = \{G'_n[\Omega_n(\hat{\theta})]^{-1}G_n(I_k - \widehat{D}_n)\}^{-1}\Sigma_n(\hat{\theta}, \Omega_n(\hat{\theta}))\{G'_n[\Omega_n(\hat{\theta})]^{-1}G_n(I_k - \widehat{D}_n)\}^{-1'}$$

where $\Sigma_n(\hat{\theta}, \Omega_n(\hat{\theta}))$ is defined in (23) and $\widehat{D}_n$ is evaluated at $\hat{\theta}$. Not surprisingly, this formula is identical to Hansen and Lee (2020)'s misspecification-robust variance estimator for iterated GMM. The finite sample corrected formula suggested by Windmeijer (2000) is

$$\widehat{V}_w(\hat{\theta}) = (I_k - \widehat{D}_n)^{-1} \left(G'_n[\Omega_n(\hat{\theta})]^{-1}G_n \right)^{-1} (I_k - \widehat{D}_n)^{-1'}$$

On the other hand, a similar finite sample correction may not be obtained for CU GMM. Windmeijer (2005) showed that if the derivative of the moment function is a function of the parameter, then the proposed formula would not necessarily give finite sample corrections. The same argument applied to CU GMM. Let $\hat{\theta}$ be the CU GMM estimator. For simplicity, let $k = 1$ so that θ is scalar. The FOC is

$$0 = \left(G_n - \frac{1}{2}g_n(\hat{\theta})'[\Omega_n(\hat{\theta})]^{-1} \frac{\partial \Omega_n(\theta)}{\partial \theta} \Big|_{\theta=\hat{\theta}} \right)' [\Omega_n(\hat{\theta})]^{-1}g_n(\hat{\theta}).$$

This shows that even when the moment function is linear, the effective Jacobian term in the FOC still depends on the parameter. Thus, the misspecification-robust variance formula for CU GMM does not necessarily provide a finite sample correction under correct specification. Since GEL estimators have similar non-linear FOC even with linear moment functions, we expect similar conclusions.

6 Examples

6.1 Cross-sectional IV

Consider the linear IV model $y_i = X_i'\theta + e_i$ with the moment conditions $E[Z_ie_i] = 0$. The two-stage least squares (2SLS) estimator is given by

$$\hat{\theta}_1 = (X'Z(Z'Z)^{-1}Z'X)^{-1}X'Z(Z'Z)^{-1}Z'Y,$$

where $Y = [y_1, \dots, y_n]'$, $X = [X_1, \dots, X_n]'$, and $Z = [Z_1, \dots, Z_n]'$ are $n \times 1$, $n \times k$, and $n \times q$ data matrices. Using the 2SLS as the preliminary estimator, the two-step efficient GMM estimator is given by

$$\hat{\theta}_2 = (X'Z\widehat{\Omega}_1^{-1}Z'X)^{-1}X'Z\widehat{\Omega}_1^{-1}Z'Y,$$

where $\widehat{\Omega}_1 = \frac{1}{n} \sum_{i=1}^n Z_i Z_i' \hat{e}_{1i}^2$, $\hat{e}_{1i} = y_i - X_i' \hat{\theta}_1$. Also define $\hat{e}_{2i} = y_i - X_i' \hat{\theta}_2$ and the $n \times 1$ residual vector $\hat{e}_j = Y - X \hat{\theta}_j$ for $j = 1, 2$.

The doubly corrected variance estimators of the 2SLS and two-step GMM are

$$\begin{aligned}\widehat{V}_{dc}(\hat{\theta}_1) &= \left(\frac{1}{n} X' Z (Z' Z)^{-1} Z' X \right)^{-1} \frac{1}{n} \sum_{i=1}^n \hat{m}_{1i} \hat{m}'_{1i} \left(\frac{1}{n} X' Z (Z' Z)^{-1} Z' X \right)^{-1}, \\ \widehat{V}_{dc}(\hat{\theta}_2) &= \widehat{V}(\hat{\theta}_2) + \widehat{D}_n \widehat{C}(\hat{\theta}_1, \hat{\theta}_2) + \widehat{C}(\hat{\theta}_1, \hat{\theta}_2)' \widehat{D}'_n + \widehat{D}_n \widehat{V}_{dc}(\hat{\theta}_1) \widehat{D}'_n,\end{aligned}$$

where

$$\begin{aligned}\widehat{V}(\hat{\theta}_2) &= \left(\frac{1}{n^2} X' Z \widehat{\Omega}_1^{-1} Z' X \right)^{-1} \left(\frac{1}{n} \sum_{i=1}^n \hat{m}_{2i} \hat{m}'_{2i} \right) \left(\frac{1}{n^2} X' Z \widehat{\Omega}_1^{-1} Z' X \right)^{-1}, \\ \widehat{C}(\hat{\theta}_1, \hat{\theta}_2) &= \left(\frac{1}{n} X' Z (Z' Z)^{-1} Z' X \right)^{-1} \left(\frac{1}{n} \sum_{i=1}^n \hat{m}_{1i} \hat{m}'_{2i} \right) \left(\frac{1}{n^2} X' Z \widehat{\Omega}_1^{-1} Z' X \right)^{-1}, \\ \widehat{D}_n &= \frac{2}{n} \left(X' Z \widehat{\Omega}_1^{-1} Z' X \right)^{-1} X' Z \widehat{\Omega}_1^{-1} \sum_{i=1}^n Z_i \left(\hat{e}_{1i} Z'_i \widehat{\Omega}_1^{-1} Z' \hat{e}_2 \right) X'_i, \\ \hat{m}_{1i} &= X' Z (Z' Z)^{-1} Z_i \hat{e}_{1i} + X_i Z'_i (Z' Z)^{-1} Z' \hat{e}_1 - X' Z (Z' Z)^{-1} Z_i Z'_i (Z' Z)^{-1} Z' \hat{e}_1, \\ \hat{m}_{2i} &= \frac{1}{n} X' Z \widehat{\Omega}_1^{-1} Z_i \hat{e}_{2i} + \frac{1}{n} X_i Z'_i \widehat{\Omega}_1^{-1} Z' \hat{e}_2 - \frac{1}{n^2} X' Z \widehat{\Omega}_1^{-1} Z_i Z'_i \hat{e}_{1i}^2 \widehat{\Omega}_1^{-1} Z' \hat{e}_2.\end{aligned}$$

It is worth observing that the doubly corrected variance estimator $\widehat{V}_{dc}(\hat{\theta}_2)$ reduces to the Windmeijer corrected one $\widehat{V}_w(\hat{\theta}_2)$ if (i) the last two terms in $\hat{m}_{2i}$ and $\hat{m}_{1i}$ are ignored and (ii) $\hat{e}_{1i}$ replaces $\hat{e}_{2i}$ in $\hat{m}_{2i}$. By (i) and (ii), the variance estimators $\widehat{V}(\hat{\theta}_2)$ and $\widehat{V}_{dc}(\hat{\theta}_1)$ reduce to conventional ones $\widehat{V}(\hat{\theta}_2)$ and $\widehat{V}(\hat{\theta}_1)$, and $\widehat{C}(\hat{\theta}_1, \hat{\theta}_2)$ becomes $\widehat{V}(\hat{\theta}_2)$. In general, however, $\widehat{V}_{dc}(\hat{\theta}_2) \neq \widehat{V}_w(\hat{\theta}_2)$ because $Z' \hat{e}_j \neq 0$ for $j = 1, 2$, so the last two terms of $\hat{m}_{ji}$ are non-zero. Furthermore, it is critical (and reasonable) to use $\hat{e}_{2i}$ in $\hat{m}_{2i}$ to get robustness under misspecification.

The iterated GMM estimator is obtained as follows. Let $\hat{\theta}_0$ be any initial value. The s -step GMM estimator for $s \geq 1$ is given by

$$\hat{\theta}_s = (X' Z \widehat{\Omega}_{s-1}^{-1} Z' X)^{-1} X' Z \widehat{\Omega}_{s-1}^{-1} Z' Y,$$

where

$$\widehat{\Omega}_{s-1} = \frac{1}{n} \sum_{i=1}^n Z_i Z'_i (y_i - X'_i \hat{\theta}_{s-1})^2.$$

We iterate the s -step GMM estimator until convergence given a preset tolerance ϵ , i.e. $\|\hat{\theta}_s - \hat{\theta}_{s-1}\| < \epsilon$ to obtain the iterated GMM estimator $\hat{\theta}$. The residuals are $\hat{e}_i = y_i - X'_i \hat{\theta}$. Also let $\hat{e} = Y - X \hat{\theta}$ be the $n \times 1$ residual vector.

The doubly corrected variance estimator is

$$\begin{aligned}\widehat{V}_{dc}(\hat{\theta}) &= \widehat{H}^{-1} \left(\frac{1}{n} \sum_{i=1}^n \hat{m}_i \hat{m}_i' \right) \widehat{H}^{-1'}, \\ \widehat{H} &= \frac{1}{n^2} X' Z \widehat{\Omega}^{-1} Z' X - \frac{2}{n^3} X' Z \widehat{\Omega}^{-1} \sum_{i=1}^n Z_i \left(\hat{e}_i Z_i' \widehat{\Omega}^{-1} Z' \hat{e} \right) X_i', \\ \hat{m}_i &= \frac{1}{n} X' Z \widehat{\Omega}^{-1} Z_i \hat{e}_i + \frac{1}{n} X_i Z_i' \widehat{\Omega}^{-1} Z' \hat{e} - \frac{1}{n^2} X' Z \widehat{\Omega}^{-1} Z_i Z_i' \hat{e}_i^2 \widehat{\Omega}^{-1} Z' \hat{e}.\end{aligned}$$

In comparison, the Windmeijer corrected and the conventional variance estimators are

$$\begin{aligned}\widehat{V}_w(\hat{\theta}) &= \widehat{H}^{-1} \left(\frac{1}{n^2} X' Z \widehat{\Omega}^{-1} Z' X \right) \widehat{H}^{-1'}, \\ \widetilde{V}(\hat{\theta}) &= \left(\frac{1}{n^2} X' Z \widehat{\Omega}^{-1} Z' X \right)^{-1}.\end{aligned}$$

6.2 A Panel Data Model

Consider a panel data model with a scalar regressor

$$y_{it} = x_{it}\beta + \eta_i + v_{it}, \quad (34)$$

for $i = 1, \dots, N$ and $t = 1, \dots, T$ where η_i is the unobserved individual effects, the unknown parameter of interest is β , and the single regressor x_{it} is predetermined with respect to v_{it} (possibly including lags of the dependent variable), i.e., $E(x_{it}v_{is}) = 0$ for all $s \geq t$. After first-differencing,

$$\Delta y_{it} = \Delta x_{it}\beta + \Delta v_{it}, \quad t = 2, \dots, T,$$

the standard approach to estimate β is first differenced GMM (Arellano and Bond (1991) estimator) with the moment conditions $E(Z_i' \Delta v_i) = 0$ where Z_i is the $(T-1)$ by $T(T-1)/2$ instrument matrix

$$Z_i = \text{diag}(z'_{i2}, \dots, z'_{iT})$$

with all possible lagged instruments $z_{it} = (x_{i1}, \dots, x_{it-1})'$ for $2 \leq t \leq T$ and $\Delta v_i = (\Delta v_{i2}, \dots, \Delta v_{iT})'$. The total number of observations is $n = N(T-1)$.

Our doubly corrected variance estimator can be used for the model (34) with additional strictly exogenous, predetermined, or endogenous variables as well as the system GMM estimator (Arellano and Bover (1995) and Blundell and Bond (1998)) by stacking and modifying additional moment conditions into the instrument sets Z_i . If the panel is unbalanced, the instrument matrix can be constructed as described in Arellano and Bond (1991).

Using the initial weight matrix $\widehat{W} = n^{-1} \sum_{i=1}^N Z_i' H Z_i$, where H is a matrix with 2's on the main diagonal, -1's on the first off-diagonals and zeros elsewhere, the one-step GMM estimator is

given by

$$\hat{\beta}_1 = (\Delta X' Z \widehat{W}^{-1} Z' \Delta X)^{-1} \Delta X' Z \widehat{W}^{-1} Z' \Delta Y,$$

where $Z = (Z'_1, \dots, Z'_N)'$ is the instrument matrix, $\Delta Y = (\Delta y'_1, \dots, \Delta y'_N)'$, $\Delta X = (\Delta x'_1, \dots, \Delta x'_N)'$, $\Delta y_i = (\Delta y_{i2}, \dots, \Delta y_{iT})'$, and $\Delta x_i = (\Delta x_{i2}, \dots, \Delta x_{iT})'$. Note that scaling the weight matrix does not affect the estimator. The doubly corrected variance estimator of $\hat{\beta}_1$ is given by

$$\begin{aligned} \widehat{V}_{dc}(\hat{\beta}_1) &= n^2 \left(\Delta X' Z \widehat{W}^{-1} Z' \Delta X \right)^{-1} \left(\frac{1}{n} \sum_{i=1}^N \hat{m}_{1i} \hat{m}'_{1i} \right) \left(\Delta X' Z \widehat{W}^{-1} Z' \Delta X \right)^{-1}, \\ \hat{m}_{1i} &= \Delta X' Z \widehat{W}^{-1} Z'_i \Delta \hat{v}_{1i} + \Delta x'_i Z_i \widehat{W}^{-1} Z' \Delta \hat{v}_1 - \frac{1}{n} \Delta X' Z \widehat{W}^{-1} Z'_i H Z_i \widehat{W}^{-1} Z' \Delta \hat{v}_1, \end{aligned}$$

where $\Delta \hat{v}_{1i} = \Delta y_i - \Delta x_i \hat{\beta}_1$ and $\Delta \hat{v}_1 = (\Delta \hat{v}'_{11}, \dots, \Delta \hat{v}'_{1N})'$. The doubly corrected standard error is obtained by taking the diagonal elements of $\sqrt{\widehat{V}_{dc}(\hat{\beta}_1)/n}$. In comparison, the conventional variance estimator is given by

$$\tilde{V}(\hat{\beta}_1) = n^2 \left(\Delta X' Z \widehat{W}^{-1} Z' \Delta X \right)^{-1} \Delta X' Z \widehat{W}^{-1} \widehat{\Omega}_1 \widehat{W}^{-1} Z' \Delta X \left(\Delta X' Z \widehat{W}^{-1} Z' \Delta X \right)^{-1},$$

where

$$\widehat{\Omega}_1 = \frac{1}{n} \sum_{i=1}^N Z'_i \Delta \hat{v}_{1i} \Delta \hat{v}'_{1i} Z_i.$$

Next, consider the two-step efficient GMM estimator

$$\hat{\beta}_2 = (\Delta X' Z \widehat{\Omega}_1^{-1} Z' \Delta X)^{-1} \Delta X' Z \widehat{\Omega}_1^{-1} Z' \Delta Y.$$

Let $\Delta \hat{v}_{2i} = \Delta y_i - \Delta x_i \hat{\beta}_2$ and $\Delta \hat{v}_2 = (\Delta \hat{v}'_{21}, \dots, \Delta \hat{v}'_{2N})'$. The doubly corrected variance estimator of $\hat{\beta}_2$ is given by

$$\widehat{V}_{dc}(\hat{\beta}_2) = \widehat{V}(\hat{\beta}_2) + \widehat{D}_n \widehat{C}(\hat{\beta}_1, \hat{\beta}_2) + \widehat{C}(\hat{\beta}_1, \hat{\beta}_2)' \widehat{D}'_n + \widehat{D}_n \widehat{V}_{dc}(\hat{\beta}_1) \widehat{D}'_n,$$

where

$$\begin{aligned} \widehat{V}(\hat{\beta}_2) &= n^2 \left(\Delta X' Z \widehat{\Omega}_1^{-1} Z' \Delta X \right)^{-1} \left(\frac{1}{n} \sum_{i=1}^N \hat{m}_{2i} \hat{m}'_{2i} \right) \left(\Delta X' Z \widehat{\Omega}_1^{-1} Z' \Delta X \right)^{-1}, \\ \widehat{C}(\hat{\beta}_1, \hat{\beta}_2) &= n^2 \left(\Delta X' Z \widehat{W}^{-1} Z' \Delta X \right)^{-1} \left(\frac{1}{n} \sum_{i=1}^N \hat{m}_{1i} \hat{m}'_{2i} \right) \left(\Delta X' Z \widehat{\Omega}_1^{-1} Z' \Delta X \right)^{-1}, \\ \hat{m}_{2i} &= \Delta X' Z \widehat{\Omega}_1^{-1} Z'_i \Delta \hat{v}_{2i} + \Delta x'_i Z_i \widehat{\Omega}_1^{-1} Z' \Delta \hat{v}_2 - \frac{1}{n} \Delta X' Z \widehat{\Omega}_1^{-1} Z'_i \Delta \hat{v}_{1i} \Delta \hat{v}'_{1i} Z_i \widehat{\Omega}_1^{-1} Z' \Delta \hat{v}_2, \\ \widehat{D}_n &= \left(\Delta X' Z \widehat{\Omega}_1^{-1} Z' \Delta X \right)^{-1} \Delta X' Z \widehat{\Omega}_1^{-1} \\ &\quad \times \frac{1}{n} \sum_{i=1}^N \left(Z'_i \Delta x_i \left(\Delta \hat{v}'_2 Z \widehat{\Omega}_1^{-1} Z'_i \Delta \hat{v}_{1i} \right) + (Z'_i \Delta \hat{v}_{1i}) \left(\Delta \hat{v}'_2 Z \widehat{\Omega}_1^{-1} Z'_i \Delta x_i \right) \right). \end{aligned}$$

The doubly corrected standard error is obtained by taking the diagonal elements of $\sqrt{\widehat{V}_{dc}(\hat{\beta}_2)/n}$. Note that the Windmeijer corrected variance estimator is

$$\widehat{V}_w(\hat{\beta}_2) = \widetilde{V}(\hat{\beta}_2) + \widehat{D}_n \widetilde{V}(\hat{\beta}_2) + \widetilde{V}(\hat{\beta}_2) \widehat{D}_n' + \widehat{D}_n \widetilde{V}(\hat{\beta}_1) \widehat{D}_n',$$

where

$$\widetilde{V}(\hat{\beta}_2) = n^2 \left(\Delta X' Z \widehat{\Omega}_1^{-1} Z' \Delta X \right)^{-1}.$$

Finally, the iterated GMM estimator is given as follows. Let $\hat{\beta}_0$ be any initial value. The s -step GMM estimator for $s \geq 1$ is given by

$$\hat{\beta}_s = (\Delta X' Z \widehat{\Omega}_{s-1}^{-1} Z' \Delta X)^{-1} \Delta X' Z \widehat{\Omega}_{s-1}^{-1} Z' \Delta Y,$$

where

$$\widehat{\Omega}_{s-1} = \frac{1}{n} \sum_{i=1}^N Z_i' (\Delta y_i - \Delta x_i \hat{\beta}_{s-1}) (\Delta y_i - \Delta x_i \hat{\beta}_{s-1})' Z_i.$$

We iterate the s -step GMM estimator until convergence given a preset tolerance ϵ , i.e. $\|\hat{\beta}_s - \hat{\beta}_{s-1}\| < \epsilon$ to obtain the iterated GMM estimator $\hat{\beta}$. The residuals are $\Delta \hat{v}_i = \Delta y_i - \Delta x_i \hat{\beta}$. Also let $\Delta \hat{v} = (\Delta \hat{v}_1', \dots, \Delta \hat{v}_N')'$ be the $n \times 1$ residual vector.

The doubly corrected variance estimator for iterated GMM is given by

$$\begin{aligned} \widehat{V}_{dc}(\hat{\beta}) &= \widehat{H}^{-1} \left(\frac{1}{n} \sum_{i=1}^N \hat{m}_i \hat{m}_i' \right) \widehat{H}^{-1'}, \\ \widehat{H} &= \frac{1}{n^2} \Delta X' Z \widehat{\Omega}^{-1} Z' \Delta X \\ &\quad - \frac{1}{n^3} \Delta X' Z \widehat{\Omega}^{-1} \left(\sum_{i=1}^N (Z_i' \Delta \hat{v}_i) \left(\Delta \hat{v}' Z \widehat{\Omega}^{-1} Z_i' \Delta x_i \right) + Z_i' \Delta x_i \left(\Delta \hat{v}' Z \widehat{\Omega}^{-1} Z_i' \Delta \hat{v}_i \right) \right), \\ \hat{m}_i &= \frac{1}{n} \Delta X' Z \widehat{\Omega}^{-1} Z_i' \Delta \hat{v}_i + \frac{1}{n} \Delta X_i' Z_i \widehat{\Omega}^{-1} Z' \Delta \hat{v} - \frac{1}{n^2} \Delta X' Z \widehat{\Omega}^{-1} Z_i' \Delta \hat{v}_i \Delta \hat{v}_i' Z_i \widehat{\Omega}^{-1} Z' \Delta \hat{v}, \end{aligned}$$

and the doubly corrected standard error is obtained by taking the diagonal elements of $\sqrt{\widehat{V}_{dc}(\hat{\beta})/n}$.

In comparison, the Windmeijer corrected and the conventional variance estimators are

$$\begin{aligned} \widehat{V}_w(\hat{\beta}) &= \widehat{H}^{-1} \left(\frac{1}{n^2} \Delta X' Z \widehat{\Omega}^{-1} Z' \Delta X \right) \widehat{H}^{-1'}, \\ \widetilde{V}(\hat{\beta}) &= \left(\frac{1}{n^2} \Delta X' Z \widehat{\Omega}^{-1} Z' \Delta X \right)^{-1}. \end{aligned}$$

7 Simulation

We investigate the finite sample performance of the doubly corrected standard errors proposed in this paper and provide a thorough comparison with the conventional and the Windmeijer corrected

ones under correct specification and misspecification. We consider three different setups: (i) a cross-sectional linear IV model with potentially invalid instruments; (ii) a linear dynamic panel model with a random coefficient; (iii) a linear dynamic panel model with possibly misspecified lag specifications. The number of Monte Carlo simulation is 100,000.

In an unreported simulation, we also investigate the performance of the estimators with the centered weight matrix (30). Since the results are similar and there is no obvious pattern of better performance of the point and variance estimators based on the centered weight matrix, they are not reported.

7.1 Cross-sectional IV

We use the following simulation design which is a simple linear instrumental variable regression with a single endogenous regressor. The model to be estimated is

$$\begin{aligned} y_i &= x_i \beta_0 + e_i, \\ E(z_i e_i) &= 0 \end{aligned} \tag{35}$$

where x_i and β_0 are scalar and $z_i = (z_{1i}, z_{2i}, z_{3i}, z_{4i})'$ is a vector of instrumental variables. We estimate β_0 by 2SLS (one-step), two-step, and iterated GMM, and calculate the conventional, the Windmeijer corrected, and the doubly corrected standard errors. Our data-generating process (DGP) is

$$\begin{aligned} y_i &= x_i \beta_0 + e_i, \\ x_i &= \pi_0 (z_{1i} + z_{2i} + z_{3i} + z_{4i}) + u_i, \\ e_i &= \frac{\alpha_0}{\sqrt{n}} (z_{1i} - z_{2i} + z_{3i} - z_{4i}) + 0.5u_i + \sqrt{1 - 0.5^2} v_i, \\ z_i &\sim N(0, I_4), \begin{pmatrix} u_i \\ v_i \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & z_{1i}^2 \end{bmatrix}\right). \end{aligned}$$

We set $\beta_0 = 1$, vary α_0 from 0 to 1 in steps of 0.2, and set the first-stage coefficient π_0 so that the first-stage $R^2 = 0.2$. We set the number of observations as $n = 50, 100, 500$.

The parameter α_0 is the extent that the exclusion condition is locally violated. At $\alpha_0 = 0$, the model is correctly specified. For $\alpha_0 \neq 0$, we find $E(z_i e_i) = (\alpha_0, -\alpha_0, \alpha_0, -\alpha_0)'/\sqrt{n} \neq 0$, so the moment condition (35) fails to hold in finite samples, but it holds asymptotically.

Means and standard deviations of one-step (2SLS), two-step, and iterated GMM are computed in Table 1. For all GMM estimators, we report means of the conventional standard errors ($se_{\hat{\beta}}$), the Windmeijer corrected standard errors ($se_w \hat{\beta}$), and the doubly corrected standard errors ($se_{dc} \hat{\beta}$).

Table 1 shows that our doubly corrected standard errors remain accurate regardless of misspecification, including correct specification case ($\alpha_0 = 0$); the means of corrected standard errors are very close to the standard deviations for all values of α_0 , especially for two-step and iterated

GMM. Simulation evidence reassures our theory that the doubly corrected standard errors not only take into account variation in the estimation of the weight matrix but also extra variation due to the non-zero sample moments in the over-identified model even under correct specification. Furthermore, our doubly corrected standard errors are the only valid one under misspecification.

The conventional standard error for the one-step GMM (2SLS) estimator is downward biased under correct specification ($\alpha_0 = 0$), and this bias increases with α_0 . As is well known, the conventional standard error for the two-step GMM estimator is severely downward biased when $\alpha_0 = 0$, and this bias also increases with α_0 . The Windmeijer corrected standard error works well under correct specification but does not fully account for additional variations when α_0 is non-zero. The result is similar for iterated GMM.

It is worth noting that the two-step and iterated GMM point estimates are sensitive to the local violation parameter α_0 . Interestingly, the point estimate becomes similar to the true value as the degree of misspecification α_0 increases. This is because, in our DGP, the local misspecification bias (which depends on α_0), and the higher-order asymptotic bias (which does not depend on α_0) have opposite signs so that it happens to cancel out each other as α_0 increases. In contrast, the one-step GMM point estimate varies little across α_0 because the local misspecification bias is zero.² This is specific to this DGP and cannot be generalized. The size of bias in the point estimate decreases as the sample size increases.

7.2 Linear Dynamic Panel Model

7.2.1 Random Coefficient

We next explore the finite sample performance of the doubly corrected standard error in the presence of heterogeneous effects (random coefficient) in dynamic panel model. We consider the AR(1) dynamic panel model of Blundell and Bond (1998). For $i = 1, \dots, N$ and $t = 1, \dots, T$,

$$y_{it} = \rho_0 y_{i,t-1} + \eta_i + \nu_{it}, \quad (36)$$

where η_i is an unobserved individual-specific effect and ν_{it} is an error term. The parameter of interest ρ_0 is estimated by the difference GMM based on a set of moment conditions:

$$E[y_{i,t-s}(\Delta y_{it} - \rho_0 \Delta y_{i,t-1})] = 0, \quad t = 3, \dots, T, \text{ and } s \geq 2, \quad (37)$$

The moment conditions are derived from taking differences of (36), and uses the lagged values of y_{it} as instruments. The number of moment conditions is $(T-1)(T-2)/2$.

The moment conditions are correctly specified if there is a unique parameter that satisfies (37). A sufficient condition for this to hold is that the model (36) coincides with the true DGP, but this is unlikely to be true. A reasonable deviation from the assumed model (36) is heterogeneity in ρ_0

²The local misspecification bias and the higher-order asymptotic bias can be calculated using the formula in Theorem 2 and 3 in Hwang, Kang, and Lee (2020).

across i . We assume the following DGP. For $i = 1, \dots, N$ and $t = 1, \dots, T$,

$$\begin{aligned} y_{it} &= \rho_i y_{i,t-1} + \eta_i + \nu_{it}, \\ \eta_i &\sim N(0, 1); \quad \rho_i \sim \Phi(\alpha_0 \eta_i); \quad \nu_{it} \sim N(0, 0.5^2), \\ y_{i1} &= \frac{\eta_i}{1 - \rho_i} + u_{i1}; \quad u_{i1} \sim N\left(0, \frac{1}{1 - \rho_i^2}\right), \end{aligned}$$

where $\Phi(z)$ is the standard normal cdf. At $\alpha_0 = 0$, the model is correctly specified and $\rho_i = \rho_0 = 0.5$. For $\alpha_0 \neq 0$, the effective moment condition model can be written as

$$\begin{aligned} E[y_{i,t-s}(\Delta y_{it} - \rho \Delta y_{i,t-1})] &= E[y_{i,t-s}(\Delta \nu_{it} + (\rho_i - \rho) \Delta y_{i,t-1})] \\ &= E[\rho_i y_{i,t-s} \Delta y_{i,t-1}] - \rho(\gamma_{s-1} - \gamma_{s-2}), \end{aligned}$$

where γ_j is the j th autocovariance. The last equation becomes zero at $\rho = E[\rho_i]$ if ρ_i is independent of the $\{y_{it}\}$ process. If this is the case, then the moment condition model is correctly specified and the estimand is $E[\rho_i]$. Otherwise in general, the moment condition model fails to hold at a single unique parameter value because each of the moment condition imposes a restriction

$$\rho = \frac{E[\rho_i y_{i,t-s} \Delta y_{i,t-1}]}{\gamma_{s-1} - \gamma_{s-2}},$$

but there is no reason that this should hold at a unique ρ for $s = 2, 3, \dots, t-1$. In the DGP, η_i and ρ_i are dependent through α_0 and a larger α_0 leads to larger heterogeneity. We vary α_0 from 0 to 0.3 in steps of 0.05. The pseudo-true value would depend on the instrument set and the value of α_0 under global misspecification. However, by varying α_0 by a small amount we try to capture local behavior of the standard errors when the pseudo-true value is close to the true value. The sample sizes are $N = 100, 500$ and $T = 4, 6$.

We report the simulation results in Tables 2 and 3, which are qualitatively similar to the IV setup. Tables 2 and 3 show that the doubly corrected standard errors approximate the standard deviation of the GMM estimators well regardless of misspecification. For two-step and iterated GMM, the doubly corrected standard errors are as accurate as the Windmeijer correction for small values of α_0 (including correct specification $\alpha_0 = 0$) but dominate the other in terms of accuracy for larger values of α_0 . The doubly corrected standard error for one-step GMM is slightly upward biased for small values of α_0 , but this bias decreases with a larger sample size $N = 500$.

7.2.2 Misspecified Lag Length

We use the baseline linear panel model of Windmeijer (2005) allowing for possible lag length misspecification. The model is

$$y_{it} = \beta_0 x_{it} + \eta_i + v_{it}, \tag{38}$$

for $i = 1, \dots, N$ and $t = 1, \dots, T$. The unknown parameter of interest is β_0 , and the regressor x_{it} is predetermined with respect to v_{it} , i.e., $E(x_{it}v_{it+s}) = 0$ for $s = 0, \dots, T - t$. We use the first differenced GMM estimator. The number of moment conditions is $T(T - 1)/2$ as in Section 6.2.

The DGP is

$$\begin{aligned} y_{it} &= \beta_0 x_{it} + \alpha_0 x_{it-1} + \eta_i + v_{it}, \\ x_{it} &= 0.5x_{it-1} + \eta_i + 0.5v_{it-1} + \epsilon_{it}, \\ \eta_i &\sim N(0, 1), \quad \epsilon_{it} \sim N(0, 1), \\ v_{it} &= \delta_i \tau_t \omega_{it}, \quad \omega_{it} \sim \chi_1^2 - 1, \\ \delta_i &\sim \text{Uniform}[0.5, 1.5], \quad \tau_t = 0.5 + 0.1(t - 1). \end{aligned} \tag{39}$$

We generate initial 50 time periods with $\tau_t = 0.5$ for $t = -49, \dots, 0$ and $x_{i,-49} \sim N(\eta_i/0.5, 1/0.75)$ same as Windmeijer (2005). The parameter α_0 in (39) governs the degree of misspecification. When $\alpha_0 = 0$, the model (38) is correctly specified which reduces to that of Windmeijer (2005).

The model (38) is misspecified for $\alpha_0 \neq 0$. We discuss the pseudo-true value and its interpretation. Since $0 = E[x_{is}\Delta v_{it}]$, the effective moment condition can be written as

$$0 = E[x_{is}(\Delta y_{it} - \beta^* \Delta x_{it})] = (\beta_0 - \beta^*)E[x_{is}\Delta x_{it}] + \alpha_0 E[x_{is}\Delta x_{it-1}] \tag{40}$$

for all $1 \leq s < t$ and $2 \leq t \leq T$.

First, consider $T = 2$ where there is only one moment condition so that the model is just-identified. By setting $s = 1$ and $t = 2$, the unique solution to (40) is $\beta^* = \beta_0 - \alpha_0$. Since (40) is equal to zero at β^* , the moment condition is not misspecified, although the lag is misspecified. Since the model is just-identified $\beta^* = \beta_0 - \alpha_0$ is considered as the true value but this is not equal to β_0 unless $\alpha_0 = 0$. An implication is that model misspecification, and pseudo-true values are not only pertinent to over-identified models.

For $T \geq 3$, the model is overidentified and each of the moment condition holds at

$$\beta_{t,s}^* = \beta_0 + \alpha_0 \frac{E[x_{i,t-1}x_{is}] - E[x_{i,t-2}x_{is}]}{E[x_{it}x_{is}] - E[x_{i,t-1}x_{is}]}.$$

Note that $\beta_{t,s}^*$ can vary across t and s for a nonzero α_0 . For example, $\beta_{3,1}^* = \beta_0 + 2\alpha_0$ and $\beta_{3,2}^* = \beta_0 - \alpha_0$. The pseudo-true value β^* is a weighted average of $\beta_{t,s}^*$'s given T and the weight matrix. One can interpret that β^* represents the causal effects of past and present values of x_{it} 's on y_{it} in the true DGP.

Tables 4 and 5 report estimation results for $\beta_0 = 1$, $N = 100,500$ and $T = 4, 6$. The degree of misspecification α_0 is varied across $\{0, 0.05, 0.1, 0.2, 0.3\}$. The first column ($\alpha_0 = 0$) in Table 4 replicates Monte Carlo studies in Windmeijer (2005, Table 1). Tables 4 and 5 also report the rejection rates of the J test with the nominal size of 5%. Although the rejection rate increases as the degree of misspecification increases and the sample size increase, the J test tends to not

correctly detect the violation of the overidentifying moment restrictions in small samples.

The implication of the results in Tables 4 and 5 are largely unchanged as in two previous simulation experiments; doubly corrected standard errors approximate the standard deviations well regardless of model misspecification. In this simulation experiment, the Windmeijer correction works best under correct specification but becomes downward biased as α_0 increases. Note that deviation from correct specification makes the bias of the conventional standard error and the Windmeijer corrected standard error larger, and this bias does not disappear with a larger sample size of $N = 500$.

7.3 Size

This section investigates the small sample performance of the t tests with the proposed standard errors. We report the size of the t tests under correct specification for the simulation setups considered in previous sections.

Based on the one-step, two-step, and iterated GMM estimators, Table 6 evaluates the size of the t tests for nominal size 5% using various standard errors. In the column labeled t , we report the size of the test based on conventional heteroskedasticity-robust standard errors. t_w is based on the Windmeijer corrected standard errors, while t_{dc} is based on the doubly corrected standard errors in this paper. Further, we report the size of the bootstrap t test using the doubly corrected standard errors (t_{dc-bs}), which is equivalent to the misspecification-robust (MR) bootstrap of Lee (2014). We calculate the bootstrap critical values based on 1,000 bootstrap replications per Monte Carlo replication. Results for larger sample sizes ($n = 500$ in Table 1, Table 3 and 5) are not reported here for brevity as the results are similar across different methods.

For Setups 1 and 2, tests using the conventional standard errors (t) are severely oversized. Using the Windmeijer corrected standard errors (t_w), and the doubly corrected standard errors (t_{dc}) for the two-step and iterated estimator improve the size and perform similarly, although both are moderately oversized. The size distortion decreases when we increase the sample size. Using the MR bootstrap with the doubly corrected standard errors (t_{dc-bs}) improves the size of the test dramatically. The excellent size property of the MR bootstrap t test is theoretically justified by its asymptotic refinements, which is formally shown by Lee (2014). Perhaps surprisingly, using the doubly corrected standard errors for the t test and bootstrap t test improves size properties considerably for the one-step estimator where the Windmeijer correction is not available.

For Setup 3, t based on the one-step estimator and t_w with the two-step estimator have good size properties, and the same results can be found in Windmeijer (2005, Figure 1). t_{dc} has similar size properties to t_w , but is slightly undersized for the one-step estimator and slightly oversized for the two-step and iterated estimators. The MR bootstrap test is slightly undersized. Windmeijer (2005) and Bond and Windmeijer (2005) report similar results for the nonparametric bootstrap test of Hall and Horowitz (1996) based on the two-step estimator and explain that the performance of the bootstrap deteriorates with an increasing number of moment conditions. Since t_{dc} becomes oversized with the number of moment conditions, this suggests that some components of the doubly

corrected variance estimator may be sensitive to the number of moment conditions. This issue deserves further future investigation.

8 Conclusion

We propose doubly corrected standard errors for the over-identified one-step, two-step, and iterated GMM estimators in linear moment conditions. Our double correction considers the over-identification bias in variance estimation on top of the commonly used finite-sample correction of Windmeijer (2005), which corrects the bias from estimating the efficient weight matrix.

Monte Carlo simulations show that our doubly corrected formulas have a much more accurate variance estimator and result in size accuracy than conventional ones regardless of the model misspecification. Our formulas do not entail much extra computational cost because their main ingredients are traditional GMM estimators using original data. Thus we strongly recommend that practitioners use our formulas in linear GMM models, including linear IV and dynamic panel regression.

Appendix: Proofs

In the proofs, “LLN” refers to the weak law of large numbers, “CLT” refers to the multivariate Lindeberg-Lévy central limit theorem (e.g. Theorem 6.3. of Hansen, 2020) and “CMT” refers to the continuous mapping theorem.

Lemma 1 is on geometric expansion of a matrix. It builds on Corollary 1 of Magdalinos (1992).

Lemma 1. *Let X_n and Y_n be square random matrices. If X_n^{-1} and $(X_n + Y_n/\sqrt{n})^{-1}$ exist and X_n^{-1} and Y_n are of order $O_p(1)$, then following holds for any nonnegative integer q ,*

$$\left(X_n + \frac{1}{\sqrt{n}}Y_n\right)^{-1} = \sum_{j=0}^q \left(-\frac{1}{\sqrt{n}}X_n^{-1}Y_n\right)^j X_n^{-1} + O_p(n^{-(q+1)/2}).$$

Proof of Lemma 1: Let $S_n = X_n^{-1}Y_n$ and consider the following identity,

$$\sum_{j=0}^q \left(-\frac{1}{\sqrt{n}}\right)^j S_n^j \left(I + \frac{1}{\sqrt{n}}S_n\right) = I - \left(-\frac{1}{\sqrt{n}}\right)^{q+1} S_n^{q+1}.$$

Using $I + n^{-1/2}S_n = X_n^{-1}(X_n + n^{-1/2}Y_n)$, $S_n = X_n^{-1}Y_n$ and rearranging terms, we have

$$\begin{aligned} \left(X_n + \frac{1}{\sqrt{n}}Y_n\right)^{-1} &= \sum_{j=0}^q \left(-\frac{1}{\sqrt{n}}\right)^j S_n^j X_n^{-1} + \left(-\frac{1}{\sqrt{n}}\right)^{q+1} S_n^{q+1} \left(X_n + \frac{1}{\sqrt{n}}Y_n\right)^{-1} \\ &= \sum_{j=0}^q \left(-\frac{1}{\sqrt{n}}\right)^j (X_n^{-1}Y_n)^j X_n^{-1} + \left(-\frac{1}{\sqrt{n}}\right)^{q+1} O_p(1) \end{aligned}$$

by the assumptions of the lemma. □

Let

$$\begin{aligned}\frac{\partial \Omega(\theta)}{\partial \theta_{[j]}} &= E \left[\frac{\partial \Omega_n(\theta)}{\partial \theta_{[j]}} \right], \\ \frac{\partial^2 \Omega(\theta)}{\partial \theta_{[j]} \partial \theta_{[l]}} &= E \left[\frac{\partial^2 \Omega_n(\theta)}{\partial \theta_{[j]} \partial \theta_{[l]}} \right].\end{aligned}$$

This is a slight abuse of notation because the LHS does not mean differentiation of the expectation. Lemma 2 establishes some useful convergence results.

Lemma 2. *Under Assumption 1, the followings hold:*

$$g_n(\theta_1) - g_1 = O_p(n^{-1/2}), \quad (41)$$

$$g_n(\theta_2) - g_2 = O_p(n^{-1/2}), \quad (42)$$

$$G_n - G = O_p(n^{-1/2}), \quad (43)$$

$$W_n - W = O_p(n^{-1/2}), \quad (44)$$

$$\Omega_n(\theta_1) - \Omega_1 = O_p(n^{-1/2}), \quad (45)$$

$$W_n^{-1} - W^{-1} = O_p(n^{-1/2}), \quad (46)$$

$$[\Omega_n(\theta_1)]^{-1} - [\Omega_1]^{-1} = O_p(n^{-1/2}), \quad (47)$$

and for each j , l th element of θ ,

$$\frac{\partial \Omega_n(\theta_1)}{\partial \theta_{[j]}} - \frac{\partial \Omega(\theta_1)}{\partial \theta_{[j]}} = O_p(n^{-1/2}), \quad (48)$$

$$\frac{\partial^2 \Omega_n(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]}} - \frac{\partial^2 \Omega(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]}} = O_p(n^{-1/2}), \quad (49)$$

In addition, if $\hat{\theta}_1 - \theta_1 = O_p(n^{-1/2})$, then

$$\Omega_n(\hat{\theta}_1) - \Omega_1 = O_p(n^{-1/2}), \quad (50)$$

$$[\Omega_n(\hat{\theta}_1)]^{-1} - [\Omega_1]^{-1} = O_p(n^{-1/2}). \quad (51)$$

Proof of Lemma 2: Under Assumption 1 (ii) and (v), (41)-(45) immediately follow by CLT.

Since $g(X_i, \theta)$ is linear in θ ,

$$\begin{aligned}\frac{\partial \Omega_n(\theta_1)}{\partial \theta_{[j]}} &= \frac{1}{n} \sum_{i=1}^n g(X_i, \theta_1) \frac{\partial g(X_i, \theta_1)'}{\partial \theta_{[j]}} + \frac{1}{n} \sum_{i=1}^n \frac{\partial g(X_i, \theta_1)}{\partial \theta_{[j]}} g(X_i, \theta_1)', \\ \frac{\partial^2 \Omega_n(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]}} &= \frac{1}{n} \sum_{i=1}^n \frac{\partial g(X_i, \theta_1)}{\partial \theta_{[l]}} \frac{\partial g(X_i, \theta_1)'}{\partial \theta_{[j]}} + \frac{1}{n} \sum_{i=1}^n \frac{\partial g(X_i, \theta_1)}{\partial \theta_{[j]}} \frac{\partial g(X_i, \theta_1)'}{\partial \theta_{[l]}}.\end{aligned}$$

Under Assumption 1 (ii) and (v), we apply CLT and the Cauchy-Schwarz inequality to have (48) and (49).

Next we show (50). Observing that the third derivative of $\Omega_n(\theta)$ with respect to θ equals to zero, we use the second-order Taylor expansion to obtain

$$\begin{aligned}\Omega_n(\hat{\theta}_1) &= \Omega_1 + (\Omega_n(\theta_1) - \Omega_1) + \sum_j \frac{\partial \Omega(\theta_1)}{\partial \theta_{[j]}} (\hat{\theta}_{1[j]} - \theta_{1[j]}) + \sum_j \left(\frac{\partial \Omega_n(\theta_1)}{\partial \theta_{[j]}} - \frac{\partial \Omega(\theta_1)}{\partial \theta_{[j]}} \right) (\hat{\theta}_{1[j]} - \theta_{1[j]}) \\ &\quad + \frac{1}{2} \sum_j \sum_l \frac{\partial^2 \Omega(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]}} (\hat{\theta}_{1[j]} - \theta_{1[j]}) (\hat{\theta}_{1[l]} - \theta_{1[l]}) \\ &\quad + \frac{1}{2} \sum_j \sum_l \left(\frac{\partial^2 \Omega_n(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]}} - \frac{\partial^2 \Omega(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]}} \right) (\hat{\theta}_{1[j]} - \theta_{1[j]}) (\hat{\theta}_{1[l]} - \theta_{1[l]}).\end{aligned}$$

By Assumption 1 (ii) and (v), the Cauchy-Schwarz inequality implies

$$\frac{\partial \Omega(\theta_1)}{\partial \theta_{[j]}} < \infty \quad \text{and} \quad \frac{\partial^2 \Omega(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]}} < \infty. \quad (52)$$

Provided that $\hat{\theta}_1 - \theta_1 = O_p(n^{-1/2})$, (50) holds by (45), (48), (49), and (52).

Finally, applying Lemma 1 with $q = 0$ to (44), (45), and (50), we obtain (46), (47), and (51), respectively. $\square$

Proof of Theorem 1: The proof consists of two parts. We first derive the asymptotic variance matrices of the one-step and two-step GMM estimators without assuming correct specification of the moment condition. Then we show that the doubly corrected variance estimators are consistent.

First, Lemma 2, CMT, and the Woodbury matrix identity together imply that

$$\begin{aligned}(G'_n W_n^{-1} G_n)^{-1} &= (G' W^{-1} G)^{-1} (I_k + (G'_n W_n^{-1} G_n - G' W^{-1} G)(G' W^{-1} G)^{-1})^{-1} \\ &= (G' W^{-1} G)^{-1} (I_k + o_p(1)),\end{aligned} \quad (53)$$

and similarly

$$(G'_n [\Omega_n(\theta_1)]^{-1} G_n)^{-1} = (G' \Omega_1^{-1} G)^{-1} (I_k + o_p(1)). \quad (54)$$

The FOC of the one-step GMM estimator is

$$0 = G'_n W_n^{-1} g_n(\hat{\theta}_1), \quad (55)$$

which holds with probability approaching zero by Assumption 1(i). By expanding (55) around the pseudo-true value θ_1 and rearranging, we have

$$\begin{aligned}\hat{\theta}_1 - \theta_1 &= - (G'_n W_n^{-1} G_n)^{-1} G'_n W_n^{-1} g_n(\theta_1) \\ &= - (G' W^{-1} G)^{-1} (I_k + o_p(1)) G'_n W_n^{-1} g_n(\theta_1).\end{aligned}$$

By Lemma 2 and the population FOC $G'W^{-1}g_1 = 0$,

$$G'_n W_n^{-1} g_n(\theta_1) = G'W^{-1}g_n(\theta_1) + G'_n W^{-1}g_1 - G'W^{-1}W_n W^{-1}g_1 + O_p(n^{-1}).$$

Let

$$m_{1i} = G'W^{-1}g(X_i, \theta_1) + G(X_i)'W^{-1}g_1 - G'W^{-1}W(X_i)W^{-1}g_1.$$

Since $Em_{1i} = 0$, by CLT under Assumption 1(v)

$$\frac{1}{\sqrt{n}} \sum_{i=1}^n m_{1i} \xrightarrow{d} N(0, Em_{1i}m'_{1i}), \quad (56)$$

as $n \rightarrow \infty$. Now we can write

$$\begin{aligned} \sqrt{n}(\hat{\theta}_1 - \theta_1) &= - (G'W^{-1}G)^{-1}(I_k + o_p(1)) \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n m_{1i} + O_p(n^{-1/2}) \right) \\ &= - (G'W^{-1}G)^{-1} \frac{1}{\sqrt{n}} \sum_{i=1}^n m_{1i} + o_p(1) \end{aligned} \quad (57)$$

$$\xrightarrow{d} N(0, V_1), \quad (58)$$

as $n \rightarrow \infty$ where V_1 is the asymptotic variance matrix of one-step GMM under misspecification given by

$$V_1 = (G'W^{-1}G)^{-1}Em_{1i}m'_{1i}(G'W^{-1}G)^{-1}.$$

The FOC of the two-step GMM estimator is

$$0 = G'_n[\Omega_n(\hat{\theta}_1)]^{-1}g_n(\hat{\theta}_2).$$

By expanding $g_n(\hat{\theta}_2)$ around θ_2 and then expanding $[\Omega_n(\hat{\theta}_1)]^{-1}$ around θ_1 ,

$$\begin{aligned} \hat{\theta}_2 - \theta_2 &= - (G'_n[\Omega_n(\hat{\theta}_1)]^{-1}G_n)^{-1}G_n[\Omega_n(\hat{\theta}_1)]^{-1}g_n(\theta_2) \\ &= - (G'_n[\Omega_n(\theta_1)]^{-1}G_n)^{-1}G_n[\Omega_n(\theta_1)]^{-1}g_n(\theta_2) + D_n^*(\hat{\theta}_1 - \theta_1) + R_n^*, \end{aligned} \quad (59)$$

where $D_n^* = F_{1n}^* + F_{2n}^*$ and R_n^* is the remainder term.

First, $F_{1n}^* = o_p(1)$ by Lemma 2 and the population FOC $G'\Omega_1^{-1}g_2 = 0$. By Lemma 2, $F_{2n}^* = D^* + o_p(1)$, where

$$D^*[\cdot, j] = (G'\Omega_1^{-1}G)^{-1}G'\Omega_1^{-1} \left. \frac{\partial \Omega(\theta)}{\partial \theta_{[j]}} \right|_{\theta=\theta_1} \Omega_1^{-1}g_2.$$

Hence we can write $D_n^* = D^* + o_p(1)$.

Next we show $R_n^* = O_p(n^{-1})$. Let

$$Q_n(\theta) = \{G'_n[\Omega_n(\theta)]^{-1}G_n\}^{-1}G'_n[\Omega_n(\theta)]^{-1}g_n(\theta_2).$$

Since the third derivative of $\Omega_n(\theta)$ w.r.t. θ equals to zero, the fifth derivative of $Q_n(\theta)$ w.r.t. θ equals to zero by the chain rule. Thus,

$$\begin{aligned} R_n &= \frac{1}{2!} \sum_j \sum_l \frac{\partial^2 Q_n(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]}} (\hat{\theta}_{1[j]} - \theta_{1[j]}) (\hat{\theta}_{1[l]} - \theta_{1[l]}) \\ &\quad + \frac{1}{3!} \sum_j \sum_l \sum_m \frac{\partial^3 Q_n(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]} \partial \theta_{[m]}} (\hat{\theta}_{1[j]} - \theta_{1[j]}) (\hat{\theta}_{1[l]} - \theta_{1[l]}) (\hat{\theta}_{1[m]} - \theta_{1[m]}) \\ &\quad + \frac{1}{4!} \sum_j \sum_l \sum_m \sum_s \frac{\partial^4 Q_n(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]} \partial \theta_{[m]} \partial \theta_{[s]}} (\hat{\theta}_{1[j]} - \theta_{1[j]}) (\hat{\theta}_{1[l]} - \theta_{1[l]}) (\hat{\theta}_{1[m]} - \theta_{1[m]}) (\hat{\theta}_{1[s]} - \theta_{1[s]}). \end{aligned}$$

By LLN under Assumption 1, $\frac{\partial^2 Q_n(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]}} = O_p(1)$, $\frac{\partial^3 Q_n(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]} \partial \theta_{[m]}} = O_p(1)$, and $\frac{\partial^4 Q_n(\theta_1)}{\partial \theta_{[j]} \partial \theta_{[l]} \partial \theta_{[m]} \partial \theta_{[s]}} = O_p(1)$. Since we have shown $\hat{\theta}_1 - \theta_1 = O_p(n^{-1/2})$, it follows that $R_n^* = O_p(n^{-1})$. Note that this also justifies the expansion (6) by setting $\theta_1 = \theta_2 = \theta_0$ and acknowledging that $g_n(\theta_0) = O_p(n^{-1/2})$ under correct specification.

By multiplying $\sqrt{n}$ on both sides (59) can now be written as

$$\sqrt{n}(\hat{\theta}_2 - \theta_2) = - (G'_1 \Omega_1^{-1} G)^{-1} (I_k + o_p(1)) G_n[\Omega_n(\theta_1)]^{-1} \sqrt{n} g_n(\theta_2) \quad (60)$$

$$+ D^* \sqrt{n}(\hat{\theta}_1 - \theta_1) + o_p(1). \quad (61)$$

Take (60). By Lemma 2 and the population FOC $G'_1 \Omega_1^{-1} g_2 = 0$,

$$G_n[\Omega_n(\theta_1)]^{-1} g_n(\theta_2) = G'_1 \Omega_1^{-1} g_n(\theta_2) + G'_n \Omega_1^{-1} g_2 - G'_1 \Omega_1^{-1} \Omega_n(\theta_1) \Omega_1^{-1} g_2 + O_p(n^{-1}). \quad (62)$$

Let

$$m_{2i} = G'_1 \Omega_1^{-1} g(X_i, \theta_2) + G(X_i)' \Omega_1^{-1} g_2 - G'_1 \Omega_1^{-1} g(X_i, \theta_1) g(X_i, \theta_1)' \Omega_1^{-1} g_2.$$

Since $Em_{2i} = 0$, by CLT under Assumption 1(v)

$$\frac{1}{\sqrt{n}} \sum_{i=1}^n \begin{pmatrix} m_{2i} \\ m_{1i} \end{pmatrix} \xrightarrow{d} N \left(0, \begin{pmatrix} Em_{2i} m'_{2i} & Em_{2i} m'_{1i} \\ Em_{1i} m'_{2i} & Em_{1i} m'_{1i} \end{pmatrix} \right),$$

as $n \rightarrow \infty$. Replacing (62) and (57) into (60) and (61), respectively, to obtain

$$\begin{aligned} \sqrt{n}(\hat{\theta}_2 - \theta_2) &= - \left[(G'_1 \Omega_1^{-1} G)^{-1} \quad D^* (G' W^{-1} G)^{-1} \right] \frac{1}{\sqrt{n}} \sum_{i=1}^n \begin{pmatrix} m_{2i} \\ m_{1i} \end{pmatrix} + o_p(1) \\ &\xrightarrow{d} N(0, V_2), \end{aligned} \quad (63)$$

as $n \rightarrow \infty$ where V_2 is the asymptotic variance matrix of two-step GMM under misspecification given by

$$V_2 = (G'\Omega_1^{-1}G)^{-1}Em_{2i}m'_{2i}(G'\Omega_1^{-1}G)^{-1} + D^*(G'W^{-1}G)^{-1}Em_{1i}m'_{2i}(G'\Omega_1^{-1}G)^{-1} \\ + (G'\Omega_1^{-1}G)^{-1}Em_{2i}m'_{1i}(G'W^{-1}G)^{-1}D^{*'} + D^*(G'W^{-1}G)^{-1}Em_{1i}m'_{1i}(G'W^{-1}G)^{-1}D^{*'}.$$

Since we have shown the asymptotic variance matrices of the GMM estimators robust to misspecification, the first part of the proof is complete.

Note that $m_{1i} = G'W^{-1}g(X_i, \theta_0)$, $m_{2i} = G'\Omega_0^{-1}g(X_i, \theta_0)$, and $D^* = 0$ under correct specification and V_1 and V_2 coincide to the conventional asymptotic variance matrices.

Next we show that $\widehat{V}_{dc}(\hat{\theta}_1) \xrightarrow{p} V_1$ and $\widehat{V}_{dc}(\hat{\theta}_2) \xrightarrow{p} V_2$. Since (53) and (54) hold, by the CMT it suffices to show

$$\frac{1}{n} \sum_{i=1}^n m_i(\hat{\theta}_2, \Omega_n(\hat{\theta}_1)) m_i(\hat{\theta}_2, \Omega_n(\hat{\theta}_1))' \xrightarrow{p} Em_{2i}m'_{2i}, \quad (64)$$

$$\frac{1}{n} \sum_{i=1}^n m_i(\hat{\theta}_1, W_n) m_i(\hat{\theta}_2, \Omega_n(\hat{\theta}_1))' \xrightarrow{p} Em_{1i}m'_{2i}, \quad (65)$$

$$\frac{1}{n} \sum_{i=1}^n m_i(\hat{\theta}_1, W_n) m_i(\hat{\theta}_1, W_n)' \xrightarrow{p} Em_{1i}m'_{1i}, \quad (66)$$

$$\widehat{D}_n \xrightarrow{p} D^*. \quad (67)$$

(64)-(66) follow if we show

$$m_i(\hat{\theta}_2, \Omega_n(\hat{\theta}_1)) = m_{2i} + O_p(n^{-1/2}), \quad (68)$$

$$m_i(\hat{\theta}_1, W_n) = m_{1i} + O_p(n^{-1/2}). \quad (69)$$

Since by Lemma 2, (58), and (63), for $j = 1, 2$,

$$g_n(\hat{\theta}_j) = g_j + (g_n(\theta_j) - g_j) + G(\hat{\theta}_j - \theta_j) + (G_n - G)(\hat{\theta}_j - \theta_j) \\ = g_j + O_p(n^{-1/2})$$

and

$$g(X_i, \hat{\theta}_j) = g(X_i, \theta_j) + G(X_i)(\hat{\theta}_j - \theta_j) = g(X_i, \theta_j) + O_p(n^{-1/2}).$$

Thus, (68) and (69) follow. To show (67), we use (42), (43), (48), (51) of Lemma 2. $\square$

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	α_0	0	0.2	0.4	0.6	0.8	1
$n = 50$	$\hat{\beta}_1$	1.0833	1.0859	1.0823	1.0833	1.0840	1.0811
	sd $\hat{\beta}_1$	0.3229	0.3226	0.3295	0.3417	0.3503	0.3746
	se $\hat{\beta}_1$	0.2962	0.2963	0.2985	0.3022	0.3047	0.3113
	se _{dc} $\hat{\beta}_1$	0.3346	0.3363	0.3417	0.3516	0.3619	0.3794
	$\hat{\beta}_2$	1.0736	1.0656	1.0517	1.0421	1.0344	1.0243
	sd $\hat{\beta}_2$	0.3029	0.3035	0.3142	0.3293	0.3470	0.3753
	se $\hat{\beta}_2$	0.2544	0.2549	0.2575	0.2616	0.2646	0.2720
	se _w $\hat{\beta}_2$	0.2889	0.2900	0.2945	0.3039	0.3133	0.3281
	se _{dc} $\hat{\beta}_2$	0.3101	0.3144	0.3231	0.3398	0.3570	0.3813
	$\hat{\beta}_{iter}$	1.0778	1.0672	1.0519	1.0390	1.0290	1.0154
	sd $\hat{\beta}_{iter}$	0.3026	0.3028	0.3147	0.3312	0.3517	0.3827
	se $\hat{\beta}_{iter}$	0.2513	0.2531	0.2573	0.2631	0.2680	0.2766
	se _w $\hat{\beta}_{iter}$	0.2850	0.2859	0.2919	0.3028	0.3140	0.3316
	se _{dc} $\hat{\beta}_{iter}$	0.3069	0.3086	0.3176	0.3340	0.3500	0.3742
$n = 100$	$\hat{\beta}_1$	1.0411	1.0408	1.0413	1.0411	1.0402	1.0412
	sd $\hat{\beta}_1$	0.2326	0.2315	0.2337	0.2373	0.2420	0.2477
	se $\hat{\beta}_1$	0.2212	0.2212	0.2218	0.2229	0.2240	0.2259
	se _{dc} $\hat{\beta}_1$	0.2354	0.2359	0.2380	0.2414	0.2458	0.2519
	$\hat{\beta}_2$	1.0353	1.0238	1.0133	1.0041	0.9940	0.9860
	sd $\hat{\beta}_2$	0.2153	0.2138	0.2187	0.2239	0.2316	0.2400
	se $\hat{\beta}_2$	0.1956	0.1957	0.1964	0.1977	0.1991	0.2010
	se _w $\hat{\beta}_2$	0.2089	0.2087	0.2099	0.2130	0.2169	0.2221
	se _{dc} $\hat{\beta}_2$	0.2135	0.2143	0.2179	0.2239	0.2315	0.2408
	$\hat{\beta}_{iter}$	1.0386	1.0260	1.0145	1.0044	0.9931	0.9836
	sd $\hat{\beta}_{iter}$	0.2143	0.2126	0.2175	0.2228	0.2311	0.2398
	se $\hat{\beta}_{iter}$	0.1946	0.1958	0.1978	0.2000	0.2024	0.2053
	se _w $\hat{\beta}_{iter}$	0.2073	0.2079	0.2101	0.2140	0.2187	0.2248
	se _{dc} $\hat{\beta}_{iter}$	0.2123	0.2129	0.2164	0.2226	0.2298	0.2392
$n = 500$	$\hat{\beta}_1$	1.0081	1.0080	1.0085	1.0080	1.0082	1.0085
	sd $\hat{\beta}_1$	0.1044	0.1048	0.1047	0.1050	0.1056	0.1061
	se $\hat{\beta}_1$	0.1035	0.1036	0.1036	0.1038	0.1037	0.1038
	se _{dc} $\hat{\beta}_1$	0.1048	0.1049	0.1050	0.1055	0.1057	0.1062
	$\hat{\beta}_2$	1.0066	1.0005	0.9949	0.9885	0.9828	0.9778
	sd $\hat{\beta}_2$	0.0962	0.0966	0.0969	0.0970	0.0981	0.0989
	se $\hat{\beta}_2$	0.0946	0.0946	0.0946	0.0948	0.0948	0.0949
	se _w $\hat{\beta}_2$	0.0958	0.0957	0.0956	0.0958	0.0958	0.0962
	se _{dc} $\hat{\beta}_2$	0.0955	0.0956	0.0958	0.0964	0.0970	0.0979
	$\hat{\beta}_{iter}$	1.0074	1.0012	0.9955	0.9891	0.9833	0.9782
	sd $\hat{\beta}_{iter}$	0.0960	0.0964	0.0966	0.0968	0.0977	0.0985
	se $\hat{\beta}_{iter}$	0.0945	0.0949	0.0951	0.0956	0.0959	0.0962
	se _w $\hat{\beta}_{iter}$	0.0957	0.0958	0.0959	0.0964	0.0966	0.0972
	se _{dc} $\hat{\beta}_{iter}$	0.0954	0.0954	0.0956	0.0962	0.0967	0.0975

Table 1: Monte Carlo Results for Linear IV: $n = 50, 100, 500$

	α_0	0	0.05	0.1	0.15	0.2	0.25	0.3
$N = 100$	$\hat{\rho}_1$	0.4256	0.4356	0.4571	0.4925	0.5397	0.5926	0.6473
$T = 4$	sd $\hat{\rho}_1$	0.3324	0.3376	0.3268	0.3173	0.3107	0.3154	0.2763
	se $\hat{\rho}_1$	0.3211	0.3215	0.3130	0.3023	0.2899	0.2775	0.2544
	se _{dc} $\hat{\rho}_1$	0.3515	0.3520	0.3431	0.3327	0.3196	0.3081	0.2815
	$\hat{\rho}_2$	0.4256	0.4350	0.4554	0.4894	0.5351	0.5876	0.6420
	sd $\hat{\rho}_2$	0.3502	0.3552	0.3443	0.3362	0.3327	0.3259	0.2966
	se $\hat{\rho}_2$	0.3113	0.3115	0.3032	0.2929	0.2806	0.2680	0.2466
	se _w $\hat{\rho}_2$	0.3376	0.3377	0.3300	0.3208	0.3087	0.2961	0.2734
	se _{dc} $\hat{\rho}_2$	0.3684	0.3673	0.3594	0.3521	0.3375	0.3258	0.3027
	$\hat{\rho}$	0.4182	0.4276	0.4467	0.4790	0.5234	0.5736	0.6267
	sd $\hat{\rho}$	0.3656	0.3702	0.3619	0.3574	0.3568	0.3551	0.3324
	se $\hat{\rho}$	0.3123	0.3122	0.3038	0.2938	0.2819	0.2688	0.2482
	se _w $\hat{\rho}$	0.3483	0.3489	0.3427	0.3350	0.3243	0.3133	0.2909
	se _{dc} $\hat{\rho}$	0.3756	0.3773	0.3695	0.3619	0.3495	0.3398	0.3122
$N = 100$	$\hat{\rho}_1$	0.4234	0.4272	0.4398	0.4626	0.4992	0.5460	0.6008
$T = 6$	sd $\hat{\rho}_1$	0.1469	0.1471	0.1468	0.1480	0.1493	0.1503	0.1477
	se $\hat{\rho}_1$	0.1458	0.1455	0.1441	0.1418	0.1377	0.1308	0.1221
	se _{dc} $\hat{\rho}_1$	0.1537	0.1540	0.1542	0.1546	0.1545	0.1509	0.1441
	$\hat{\rho}_2$	0.4217	0.4249	0.4363	0.4570	0.4909	0.5351	0.5891
	sd $\hat{\rho}_2$	0.1630	0.1640	0.1650	0.1667	0.1704	0.1727	0.1708
	se $\hat{\rho}_2$	0.1327	0.1324	0.1310	0.1284	0.1242	0.1175	0.1094
	se _w $\hat{\rho}_2$	0.1635	0.1634	0.1631	0.1626	0.1611	0.1567	0.1493
	se _{dc} $\hat{\rho}_2$	0.1628	0.1634	0.1646	0.1668	0.1693	0.1687	0.1643
	$\hat{\rho}$	0.4167	0.4194	0.4294	0.4476	0.4762	0.5137	0.5606
	sd $\hat{\rho}$	0.1782	0.1799	0.1831	0.1872	0.1972	0.2065	0.2127
	se $\hat{\rho}$	0.1328	0.1325	0.1312	0.1289	0.1250	0.1191	0.1119
	se _w $\hat{\rho}$	0.1778	0.1783	0.1794	0.1822	0.1858	0.1887	0.1887
	se _{dc} $\hat{\rho}$	0.1773	0.1784	0.1806	0.1855	0.1916	0.1965	0.1975

Table 2: Monte Carlo Results for Linear Dynamic Panel: $N = 100$ and $T = 4, 6$

	α_0	0	0.05	0.1	0.15	0.2	0.25	0.3
$N = 500$	$\hat{\rho}_1$	0.4879	0.4938	0.5170	0.5531	0.5990	0.6502	0.7016
$T = 4$	sd $\hat{\rho}_1$	0.1379	0.1363	0.1330	0.1284	0.1220	0.1131	0.1044
	se $\hat{\rho}_1$	0.1369	0.1356	0.1322	0.1265	0.1192	0.1107	0.1020
	se _{dc} $\hat{\rho}_1$	0.1389	0.1377	0.1347	0.1292	0.1222	0.1136	0.1047
	$\hat{\rho}_2$	0.4893	0.4950	0.5180	0.5539	0.6001	0.6517	0.7034
	sd $\hat{\rho}_2$	0.1400	0.1385	0.1357	0.1314	0.1254	0.1164	0.1075
	se $\hat{\rho}_2$	0.1361	0.1348	0.1315	0.1258	0.1186	0.1101	0.1014
	se _w $\hat{\rho}_2$	0.1389	0.1377	0.1349	0.1295	0.1225	0.1138	0.1046
	se _{dc} $\hat{\rho}_2$	0.1404	0.1393	0.1368	0.1318	0.1251	0.1165	0.1072
	$\hat{\rho}$	0.4891	0.4948	0.5178	0.5537	0.5999	0.6515	0.7032
	sd $\hat{\rho}$	0.1403	0.1389	0.1362	0.1319	0.1260	0.1170	0.1080
	se $\hat{\rho}$	0.1362	0.1349	0.1315	0.1259	0.1187	0.1102	0.1016
	se _w $\hat{\rho}$	0.1393	0.1382	0.1354	0.1301	0.1232	0.1145	0.1052
	se _{dc} $\hat{\rho}$	0.1408	0.1397	0.1373	0.1323	0.1256	0.1170	0.1077
$N = 500$	$\hat{\rho}_1$	0.4835	0.4869	0.4997	0.5237	0.5621	0.6130	0.6687
$T = 6$	sd $\hat{\rho}_1$	0.0691	0.0691	0.0689	0.0681	0.0676	0.0654	0.0615
	se $\hat{\rho}_1$	0.0690	0.0688	0.0680	0.0664	0.0637	0.0595	0.0544
	se _{dc} $\hat{\rho}_1$	0.0698	0.0697	0.0695	0.0691	0.0681	0.0656	0.0611
	$\hat{\rho}_2$	0.4842	0.4875	0.4998	0.5227	0.5595	0.6089	0.6639
	sd $\hat{\rho}_2$	0.0712	0.0714	0.0718	0.0723	0.0735	0.0726	0.0689
	se $\hat{\rho}_2$	0.0677	0.0675	0.0667	0.0651	0.0623	0.0581	0.0530
	se _w $\hat{\rho}_2$	0.0711	0.0711	0.0710	0.0708	0.0701	0.0678	0.0634
	se _{dc} $\hat{\rho}_2$	0.0708	0.0709	0.0713	0.0722	0.0730	0.0720	0.0682
	$\hat{\rho}$	0.4841	0.4874	0.4996	0.5223	0.5585	0.6066	0.6603
	sd $\hat{\rho}$	0.0715	0.0718	0.0723	0.0732	0.0752	0.0757	0.0737
	se $\hat{\rho}$	0.0677	0.0675	0.0667	0.0651	0.0624	0.0583	0.0534
	se _w $\hat{\rho}$	0.0715	0.0715	0.0715	0.0717	0.0720	0.0713	0.0685
	se _{dc} $\hat{\rho}$	0.0712	0.0713	0.0718	0.0730	0.0746	0.0749	0.0725

Table 3: Monte Carlo Results for Linear Dynamic Panel: $N = 500$ and $T = 4, 6$

	α_0	0	0.05	0.1	0.2	0.3
$N = 100$	$\hat{\beta}_1$	0.9793	0.9268	0.8744	0.7702	0.6647
	$T = 4$ sd $\hat{\beta}_1$	0.1521	0.1538	0.1588	0.1727	0.1925
	se $\hat{\beta}_1$	0.1469	0.1465	0.1472	0.1513	0.1589
	se _{dc} $\hat{\beta}_1$	0.1546	0.1563	0.1604	0.1744	0.1944
	J test	0.0779	0.0836	0.1127	0.2899	0.6127
	$\hat{\beta}_2$	0.9849	0.9322	0.8773	0.7587	0.6238
	sd $\hat{\beta}_2$	0.1404	0.1451	0.1552	0.1843	0.2207
	se $\hat{\beta}_2$	0.1243	0.1242	0.1253	0.1303	0.1381
	se _w $\hat{\beta}_2$	0.1390	0.1415	0.1473	0.1669	0.1919
	se _{dc} $\hat{\beta}_2$	0.1343	0.1390	0.1482	0.1775	0.2146
	J test	0.0301	0.0391	0.0767	0.2605	0.5404
	$\hat{\beta}$	0.9858	0.9334	0.8781	0.7533	0.5977
	sd $\hat{\beta}$	0.1417	0.1474	0.1600	0.2000	0.2524
	se $\hat{\beta}$	0.1243	0.1242	0.1253	0.1303	0.1381
	se _w $\hat{\beta}$	0.1393	0.1426	0.1507	0.1806	0.2230
	se _{dc} $\hat{\beta}$	0.1352	0.1406	0.1517	0.1896	0.2391
	J test	0.0295	0.0377	0.0727	0.2429	0.4983
$N = 100$	$\hat{\beta}_1$	0.9755	0.9411	0.9060	0.8368	0.7676
	$T = 6$ sd $\hat{\beta}_1$	0.1027	0.1051	0.1083	0.1172	0.1288
	se $\hat{\beta}_1$	0.1002	0.1004	0.1013	0.1037	0.1077
	se _{dc} $\hat{\beta}_1$	0.1056	0.1075	0.1107	0.1192	0.1306
	J test	0.3465	0.3533	0.4157	0.6786	0.9239
	$\hat{\beta}_2$	0.9833	0.9466	0.9080	0.8238	0.7318
	sd $\hat{\beta}_2$	0.0906	0.0948	0.1017	0.1213	0.1431
	se $\hat{\beta}_2$	0.0716	0.0720	0.0731	0.0760	0.0801
	se _w $\hat{\beta}_2$	0.0905	0.0930	0.0978	0.1117	0.1285
	se _{dc} $\hat{\beta}_2$	0.0836	0.0876	0.0944	0.1131	0.1357
	J test	0.0205	0.0277	0.0582	0.2762	0.6453
	$\hat{\beta}$	0.9857	0.9484	0.9083	0.8124	0.6885
	sd $\hat{\beta}$	0.0946	0.0998	0.1099	0.1427	0.1858
	se $\hat{\beta}$	0.0716	0.0720	0.0731	0.0760	0.0801
	se _w $\hat{\beta}$	0.0937	0.0976	0.1054	0.1320	0.1722
	se _{dc} $\hat{\beta}$	0.0866	0.0916	0.1006	0.1287	0.1675
	J test	0.0199	0.0259	0.0551	0.2553	0.5980

Table 4: Monte Carlo Results for Linear Panel Model: $N = 100$ and $T = 4, 6$

	α_0	0	0.05	0.1	0.2	0.3
$N = 500$	$\hat{\beta}_1$	0.9958	0.9406	0.8853	0.7754	0.6650
	$T = 4$ sd $\hat{\beta}_1$	0.0685	0.0695	0.0716	0.0787	0.0889
	se $\hat{\beta}_1$	0.0679	0.0677	0.0679	0.0698	0.0734
	se _{dc} $\hat{\beta}_1$	0.0686	0.0696	0.0716	0.0787	0.0889
	J test	0.0732	0.1212	0.3934	0.9524	1.0000
	$\hat{\beta}_2$	0.9970	0.9443	0.8892	0.7660	0.6225
	sd $\hat{\beta}_2$	0.0652	0.0676	0.0730	0.0891	0.1092
	se $\hat{\beta}_2$	0.0632	0.0630	0.0634	0.0657	0.0693
	se _w $\hat{\beta}_2$	0.0648	0.0662	0.0694	0.0798	0.0930
	se _{dc} $\hat{\beta}_2$	0.0634	0.0658	0.0708	0.0865	0.1060
	J test	0.0447	0.1014	0.3824	0.9426	0.9996
	$\hat{\beta}$	0.9970	0.9446	0.8897	0.7635	0.6014
	sd $\hat{\beta}$	0.0652	0.0678	0.0738	0.0946	0.1238
	se $\hat{\beta}$	0.0632	0.0630	0.0634	0.0657	0.0693
	se _w $\hat{\beta}$	0.0648	0.0662	0.0698	0.0839	0.1052
	se _{dc} $\hat{\beta}$	0.0634	0.0659	0.0715	0.0911	0.1185
	J test	0.0445	0.1004	0.3822	0.9410	0.9995
$N = 500$	$\hat{\beta}_1$	0.9947	0.9564	0.9181	0.8423	0.7662
	$T = 6$ sd $\hat{\beta}_1$	0.0473	0.0482	0.0498	0.0543	0.0600
	se $\hat{\beta}_1$	0.0469	0.0470	0.0473	0.0485	0.0504
	se _{dc} $\hat{\beta}_1$	0.0475	0.0484	0.0499	0.0543	0.0602
	J test	0.3557	0.4796	0.7820	0.9995	1.0000
	$\hat{\beta}_2$	0.9968	0.9582	0.9175	0.8276	0.7251
	sd $\hat{\beta}_2$	0.0432	0.0454	0.0494	0.0602	0.0725
	se $\hat{\beta}_2$	0.0408	0.0410	0.0414	0.0428	0.0448
	se _w $\hat{\beta}_2$	0.0431	0.0445	0.0471	0.0548	0.0640
	se _{dc} $\hat{\beta}_2$	0.0413	0.0434	0.0470	0.0573	0.0694
	J test	0.0388	0.1139	0.4500	0.9906	1.0000
	$\hat{\beta}$	0.9969	0.9584	0.9174	0.8219	0.6961
	sd $\hat{\beta}$	0.0433	0.0457	0.0504	0.0659	0.0882
	se $\hat{\beta}$	0.0408	0.0410	0.0414	0.0428	0.0448
	se _w $\hat{\beta}$	0.0431	0.0447	0.0479	0.0592	0.0770
	se _{dc} $\hat{\beta}$	0.0414	0.0437	0.0479	0.0620	0.0823
	J test	0.0387	0.1131	0.4489	0.9902	1.0000

Table 5: Monte Carlo Results for Linear Panel Model: $N = 500$ and $T = 4, 6$

	One-step			Two-step				Iterated			
	t	t_{dc}	t_{dc-bs}	t	t_w	t_{dc}	t_{dc-bs}	t	t_w	t_{dc}	t_{dc-bs}
Setup 1: Cross-sectional IV (Section 7.1)											
$n = 50$	0.109	0.084	0.072	0.142	0.112	0.110	0.077	0.151	0.123	0.122	0.071
$n = 100$	0.079	0.065	0.064	0.096	0.082	0.082	0.069	0.100	0.085	0.085	0.068
Setup 2: Dynamic Panel (Section 7.2.1)											
$N = 100, T = 4$	0.074	0.067	0.060	0.096	0.076	0.075	0.061	0.106	0.080	0.079	0.057
$N = 100, T = 6$	0.089	0.082	0.069	0.154	0.082	0.091	0.065	0.178	0.078	0.090	0.055
Setup 3: Dynamic Panel (Section 7.2.2)											
$N = 100, T = 4$	0.047	0.039	0.030	0.073	0.047	0.056	0.027	0.078	0.049	0.058	0.025
$N = 100, T = 6$	0.060	0.046	0.041	0.123	0.055	0.074	0.034	0.137	0.057	0.076	0.025

Table 6: Finite Sample Sizes of the t -test (Nominal Size 5%)